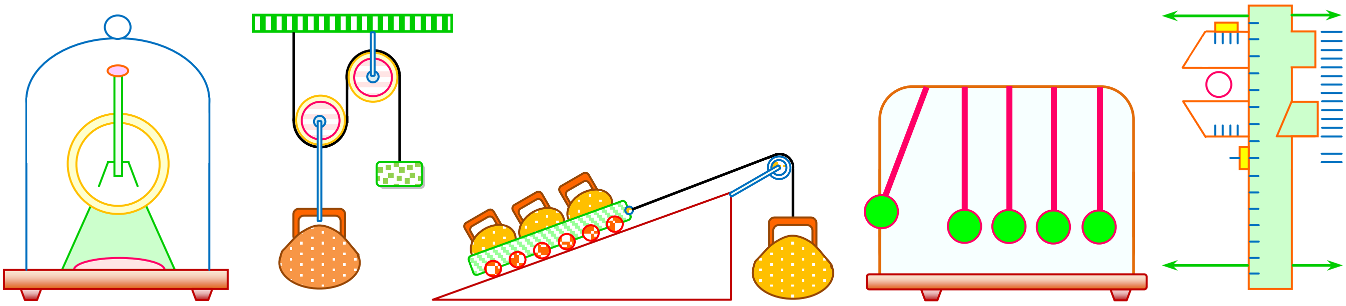


BASIC MATHEMATICS AND VECTOR

1 BASIC MATHEMATICS

1.1 ALGEBRA

(i) Quadratic Equation

$ax^2 + bx + c = 0$, where $a \neq 0$ and $a, b, c \in R$ is called a quadratic equation with real coefficients.

discriminant, $D = b^2 - 4ac$

If $D > 0 \Rightarrow$ Two roots are real and unequal

If $D = 0 \Rightarrow$ Two roots are real and equal

If $D < 0 \Rightarrow$ Two roots are complex conjugate of each other.

Roots are given by $x = \dfrac{-b \pm \sqrt{D}}{2a}$

They are denoted by α and β.

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

$ax^2 + bx + c = 0 \equiv a(x - \alpha)(x - \beta) = 0$

i.e., $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

(ii) Determinant

Determinant is a square arrangement of numbers.

They are represented by Δ or D.

e.g., $\begin{vmatrix} 4 & 3 \\ 7 & 5 \end{vmatrix}$ is a determinant of order 2 as there are two rows and two columns.

$\begin{vmatrix} 4 & -1 & 2 \\ 6 & 8 & 3 \\ 1 & 4 & 0 \end{vmatrix}$ is a determinant of order 3.

Determinants can be simplified to a single number.

The method to simplify a second order determinant is as follows

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

For 3rd order determinant:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei - hf) - b(di - gf) + c(dh - ge)$$

A determinant can be expanded along any row or any column. We have to adhere to a sign scheme.

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

For example if the same determinant is expanded along the 2nd column, we get

$$-b(di - gf) + e(ai - gc) - h(af - dc)$$

$\therefore \quad \Delta_2 = \begin{vmatrix} 4 & 3 \\ 7 & 5 \end{vmatrix} = (4)\,(5) - (7)\,(3) = 20 - 21 = -1$

$\therefore \quad \Delta_3 = \begin{vmatrix} 4 & -1 & 2 \\ 6 & 8 & 3 \\ 1 & 4 & 0 \end{vmatrix}$

(expanding along last column)

$= 2[(6)(4) - (1)(8)] - 3[(4)(4) - (1)(-1)] + 0[(4)(8) - (6)\,(-1)]$

$= 2(16) - 3(17) = 32 - 51 = -19.$

(iii) Exponential Factor

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + ...$$

is called the exponential factor.

Its value is approximately 2.718......

It is an irrational number.

It is derived from the exponential series

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ...$$

(iv) Logarithms

The equation $a^b = c$ in logarithmic notation is written as

$\log_a c = b$. $(a > 0,\ a \neq 1 \Rightarrow c > 0)$

When $a = 10$, it is called common logarithm.

When $a = e$, it is called natural logarithm. (Also written as $\ln c$)

We have the following results and rules:

(i) $\log_a 1 = 0$

(ii) $\log_a a = 1$

(iii) $\log_a(cd) = \log_a c + \log_a d$

(iv) $\log_a\left(\frac{c}{d}\right) = \log_a c - \log_a d$

(v) $\log_{a^\alpha} c^\gamma = \frac{\gamma}{\alpha}\log_a c$

(vi) $\log_a c = \frac{1}{\log_c a}$

(vii) $\log_a c = \frac{\log_k c}{\log_k a}$ (base change formula)

(viii) $a^{\log_a c} = c$

(v) Sequences and series

(a) Arithmetic sequence or progression

$a, a + d, a + 2d, a + 3d$, ...(d is the common difference)

General term, $t_k = a + (k - 1)d$ (k^{th} term)

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Arithmetic mean of two numbers a and b is defined as $A = \frac{a+b}{2}$.

(b) Geometric Progression

a, ar, ar^2, ar^3,............ (r is the common ratio)

General term, $t_k = ar^{k-1}$ (k^{th} term)

Sum to n term, $S_n = \frac{a(r^n - 1)}{r - 1}, r \neq 1$

$= na$, $r = 1$.

Sum to infinite number of terms, $S_\infty = \frac{a}{1-r}$ when $|r| < 1$

Geometric Mean of two positive numbers a and b is defined as

$G = \sqrt{ab}$.

(c) Harmonic Progression

$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \frac{1}{a+3d}$,................

General term, $t_k = \frac{1}{a+(k-1)d}$ (k^{th} term)

Harmonic Mean of two numbers a and b is defined as $H = \frac{2ab}{a+b}$.

(d) Series of Natural Numbers

$1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}$

$1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

$1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

(vi) Algebraic Identities

a) $(a + b)^2 = a^2 + b^2 + 2ab$.

b) $(a - b)^2 = a^2 + b^2 - 2ab$.

c) $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$.

d) $(a + b)^2 - (a - b)^2 = 4ab$.

e) $(a + b)^2 = (a - b)^2 + 4ab$.

f) $(a-b)^2 = (a+b)^2 - 4ab$.

g) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$.

h) $\left(\sum_{i=1}^{n} x_i\right)^2 = (x_1 + x_2 + \ldots + x_n)^2 = \sum_{i=1}^{n} x_i^2 + 2\Sigma x_i x_j, 1 \le i < j \le n$.

i) $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$.

j) $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$.

k) $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$.

l) $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$.

m) $a^2 + b^2 = (a+b)^2 - 2ab$.

n) $|a-b| = \sqrt{(a+b)^2 - 4ab}$.

(vii) Rules for Factorisation

a. $a^2 - b^2 = (a+b)(a-b)$.

b. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$.

c. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$.

d. $x^4 + x^2 + 1 = (x^2 + x + 1)(x^2 - x + 1)$.

e. $a^3 + b^3 + c^3 - 3abc = (a+b+c)[a^2 + b^2 + c^2 - ab - bc - ca]$

$= \frac{1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$.

f. $x^2 - (a+b)x + ab = (x-a)(x-b)$.

g. $x^2 + (a+b)x + ab = (x+a)(x+b)$.

h. $x^n - 1 = (x-1)(x^{n-1} + x^{n-2} + \ldots.. + x^2 + x + 1)$.

1.2 TRIGONOMETRY

(i) Degree and Radian measure of angle:

π radians = 180 degrees

(ii) Angle $= \frac{\text{Arc}}{\text{Radius}}$ *i.e.*, $\theta = \frac{s}{r}$. In this formula θ is in radians.

(iii) Signs of trigonometric ratios:

All t-ratios are positive in I[st] quadrant.
Only sine and cosecant are positive in 2[nd] quadrant.
Only tangent and cotangent are positive in 3[rd] quadrant.
Only cosine and secant are positive 4[th] quadrant.

(iv) $\sin^2 x + \cos^2 x = 1$

$\sec^2 x - \tan^2 x = 1$

$\text{cosec}^2 x - \cot^2 x = 1$

(v) $-1 \le \sin x \le 1$

$-1 \le \cos x \le 1$

$\text{cosec}\, x \ge 1$ or $\text{cosec}\, x \le -1$

$\sec x \ge 1$ or $\sec x \le -1$

(vi) $\sec x$ and $\tan x$ are not defined for $x = (2n+1)\frac{\pi}{2}, n \in \mathbf{Z}$.

$\text{cosec}\, x$ and $\cot x$ are not defined for $x = n\pi$, $n \in \mathbf{Z}$.

(vii) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A . \tan B}$

(viii) $2\sin A \cos B = \sin(A + B) + \sin(A - B)$

$2\cos A \sin B = \sin(A + B) - \sin(A - B)$

$2\cos A \cos B = \cos(A + B) + \cos(A - B)$

$2\sin A \sin B = \cos(A - B) - \cos(A + B)$

(ix) $\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

$\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

$\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

$\cos C - \cos D = 2\sin\dfrac{C+D}{2}\sin\dfrac{D-C}{2}$

(x) $\sin 2A = 2\sin A \cos A = \dfrac{2\tan A}{1+\tan^2 A}$

$$\cos 2A = \begin{cases} 1-2\sin^2 A \\ \cos^2 A - \sin^2 A = \dfrac{1-\tan^2 A}{1+\tan^2 A} \\ 2\cos^2 A - 1 \end{cases}$$

$\sin^2 A = \dfrac{1-\cos 2A}{2}$ and $\cos^2 A = \dfrac{1+\cos 2A}{2}$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

$\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

$\sin 18° = \cos 72° = \dfrac{\sqrt{5}-1}{4}$

$\cos 36° = \sin 54° = \dfrac{\sqrt{5}+1}{4}$

$\sin 72° = \cos 18° = \dfrac{\sqrt{10+2\sqrt{5}}}{4}$

$\sin 36° = \cos 54° = \dfrac{\sqrt{10-2\sqrt{5}}}{4}$

(xi) Trigonometrical Equations:

$\sin x = 0 \Leftrightarrow x = n\pi, n \in I$

$\cos x = 0 \Leftrightarrow x = (2n+1)\frac{\pi}{2}, n \in I$

$\tan x = 0 \Leftrightarrow x = n\pi, n \in I$

$\sin x = \sin\alpha \Leftrightarrow x = n\pi + (-1)^n\alpha, n \in I$

$\cos x = \cos\alpha \Leftrightarrow x = 2n\pi \pm \alpha, n \in I$

$\tan x = \tan\alpha \Leftrightarrow x = n\pi + \alpha, n \in I$

$\sin^2 x = \sin^2\alpha \Leftrightarrow x = n\pi \pm \alpha, n \in I$

$\cos^2 x = \cos^2\alpha \Leftrightarrow x = n\pi \pm \alpha, n \in I$

$\tan^2 x = \tan^2\alpha \Leftrightarrow x = n\pi \pm \alpha, n \in I$

(xii) Inverse trigonometric functions

If $\sin\theta = \frac{1}{3}$, then we write $\theta = \sin^{-1}\frac{1}{3}$

Similarly $\tan\theta = -\sqrt{3} \Rightarrow \theta = \tan^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$.

So, the angle whose trigonometric ratio is given is represented as $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$, $\text{cosec}^{-1}x$, $\sec^{-1}x$ and $\cot^{-1}x$.

(xiii) Sine rule for triangle: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

(xiv) Cosine rule for triangle: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$, *etc.*

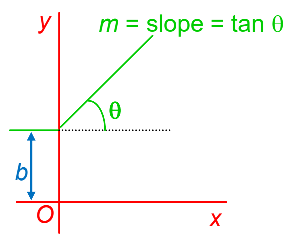
(xv) Area of triangle $= \frac{1}{2}bc\sin A$, *etc.*

$= \sqrt{s(s-a)(s-b)(s-c)}$ $\left(\text{where} \quad s = \frac{a+b+c}{2}\right)$

1.3 CO-ORDINATE GEOMETRY

(i) The distance d between two points having coordinates (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



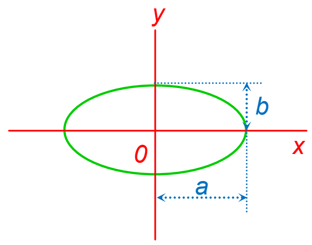
(ii) The equation of a straight line as shown in figure

$y = mx + b$

Where b is the y-intercept and m is the slope of the line.

(iii) The equation of a circle of radius R centered at the origin is

$x^2 + y^2 = R^2$



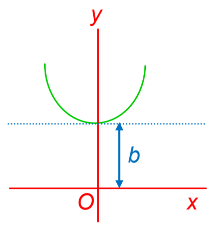
(iv) The equation of an ellipse having the origin at its centre as shown in figure.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where a is the length of the semi-major axis (the longer one) and b is the length of the semi-minor axis (the shorter one).

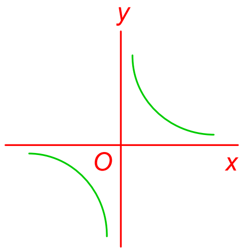
(v) The equation of a parabola the vertex of which is at $y = b$ as shown in figure. Is

$$y = ax^2 + b$$



(vi) The equation of a rectangular hyperbola as shown in figure.

$$xy = \text{constant}$$



2 CALCULUS

This is the most important tool in Mathematics and it has large number of applications in Physics and Chemistry. We shall here understand some basic concepts of Calculus. It is basically the study of functions and operations performed over them. It is split into two major divisions: Differential Calculus and Integral Calculus.

2.1 FUNCTIONS

A function denotes dependence of one quantity over the other e.g., the area enclosed by a circle depends upon its radius. We say that the area is a function of radius and it is written as

$$A = f(r)$$

where A denotes area, r denotes radius and f denotes the kind of operations to be performed on r to get A. Note that here f is not multiplied by r. It is just a symbol. The kind of dependence is $A = \pi r^2$.

On these lines if y depends on x, we write

$$y = f(x)$$

x is called independent variable and y is called the dependent variable,

e.g., $y = x^2 - 2x + 4$

$y = \sqrt{x-5}$

$y = \dfrac{1}{x+1}$, *etc.*

The dependence may be algebraic, trigonometric, logarithmic, exponential, *etc.*

e.g, $y = \tan x$

$y = \log(4 - 3x)$

$y = 2^{x+1}$

2.2 DIFFERENTIAL CALCULUS

Differential calculus is basically concerned with determination of rate of change of one quantity with respect to the quantity over which it is dependent and also with the analysis of functions.

i.e, if $y = f(x)$

We are concerned with rate of change of y w.r.t x.

If at $x = x_1$, $y = y_1$ and at $x = x_2$, $y = y_2$, then

$$\frac{\text{change in } y}{\text{change in } x} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

This is in a way average rate of change of y w.r.t x in the interval $x \in [x_1, x_2]$ or $y \in [y_1, y_2]$.

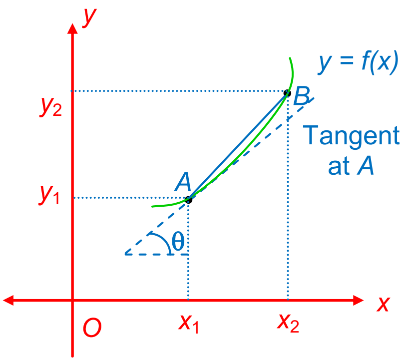
e.g., if y is interpreted as distance and x as time then this quantity gives us the average speed of the particle in that time interval. Often we are interested in determination of instantaneous speed (speed at a particular time). For that we have to assume the time interval very very small (tending to zero) so that we get the speed at that instant. When a quantity a tends to or approaches another quantity b, mathematically it is written as $a \to b$.

$\therefore$ Instantaneous rate of chage of y w.r.t. x = value of $\frac{(y_2 - y_1)}{(x_2 - x_1)}$ as $x_2 \to x_1$

It is written as $\lim_{x_2 \to x_1} \frac{(y_2 - y_1)}{(x_2 - x_1)}$ (lim stands for limit or limiting value).

The symbol $\frac{dy}{dx}$ or y' or $f'(x)$ is used to denote this value. It is also called differential coefficient or simply derivative of y w.r.t. x.

i.e., $\left.\frac{dy}{dx}\right|_{x=x_1} = \lim_{x_2 \to x_1} \frac{(y_2 - y_1)}{(x_2 - x_1)}$



From figure, it is easy to observe that $\frac{(y_2 - y_1)}{(x_2 - x_1)}$ = slope of chord AB.

As B approaches A ($x_2 \to x_1$), this chord becomes the tangent to the curve at A.

So, geometrically $\frac{dy}{dx}$ represents slope of tangent at that point.

Using the above definition, some formulae have been obtained for $\frac{dy}{dx}$ of various kinds of functions. We list them in a table below:

	Function $y = f(x)$	**Derivative $\frac{dy}{dx}$**
(i)	x^n	nx^{n-1}
(ii)	$\sin x$	$\cos x$
(iii)	$\cos x$	$-\sin x$
(iv)	$\tan x$	$\sec^2 x$
(v)	$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
(vi)	$\sec x$	$\sec x \tan x$
(vii)	$\cot x$	$-\operatorname{cosec}^2 x$
(viii)	a^x	$a^x \ln a$
(ix)	e^x	e^x
(x)	$\log_a x$	$\frac{1}{x \ln a}$
(xi)	$\ln x$	$\frac{1}{x}$

Function $y = f(x)$	Derivative $\frac{dy}{dx}$
(xii) $\sin^{-1}x$	$\frac{1}{\sqrt{1-x^2}}$
(xiii) $\cos^{-1}x$	$-\frac{1}{\sqrt{1-x^2}}$
(xiv) $\tan^{-1}x$	$\frac{1}{1+x^2}$
(xv) $\operatorname{cosec}^{-1}x$	$-\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
(xvi) $\sec^{-1}x$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
(xvii) $\cot^{-1}x$	$-\frac{1}{1+x^2}$
(xviii) c(constant)	0

Important formulae/points

- **Some rules of differentiation**

  (i) $\frac{d}{dx}\{c.f(x)\} = c.\frac{d}{dx}\{f(x)\}$, where c is a constant.

  (ii) $\frac{d}{dx}\{f(x)+g(x)\} = \frac{d}{dx}\{f(x)\} + \frac{d}{dx}\{g(x)\}$

  (iii) $\frac{d}{dx}\{f(x)-g(x)\} = \frac{d}{dx}\{f(x)\} - \frac{d}{dx}\{g(x)\}$

- **Derivative of product of two functions**

$$\frac{d}{dx}\{f(x).g(x)\} = f(x).\frac{d}{dx}\{g(x)\} + g(x)\frac{d}{dx}\{f(x)\}$$

- **Derivative of quotient of two functions**

$$\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x).\frac{d}{dx}\{f(x)\} - f(x)\frac{d}{dx}\{g(x)\}}{\{g(x)\}^2}$$

- **Derivative of function of a function**

  Chain rule : If $y = f(t)$ and $t = g(x)$, then $\frac{dy}{dx} = \left(\frac{dy}{dt}\right)\left(\frac{dt}{dx}\right)$

  This rule may be extended further.

  If $y = f(t)$, $t = g(u)$ and $u = h(x)$ then $\frac{dy}{dx} = \left(\frac{dy}{dt}\right)\left(\frac{dt}{du}\right)\left(\frac{du}{dx}\right)$

Illustration 1

Question: **Find the derivative of $(x^3 + e^x + 3^x + \cot x)$ with respect to x.**

Solution:

We have $\frac{d}{dx}(x^3 + e^x + 3^x + \cot x) = \frac{d}{dx}(x^3) + \frac{d}{dx}(e^x) + \frac{d}{dx}(3^x) + \frac{d}{dx}(\cot x)$

$= 3x^2 + e^x + 3^x(\log 3) - \operatorname{cosec}^2 x.$

Illustration 2

Question: **Differentiate : (i) $\sin x^3$ (ii) $\sin^3 x$ (iii) $e^{\sin x}$ w.r.t. x.**

Solution:

(i) Let $y = \sin x^3$.

Put $x^3 = t$, so that $y = \sin t$ and $t = x^3$.

$\therefore \quad \frac{dy}{dt} = \cos t$ and $\frac{dt}{dx} = 3x^2$.

So, $\frac{dy}{dx} = \left(\frac{dy}{dt} \times \frac{dt}{dx}\right) = 3x^2 \cos t = 3x^2 \cos x^3 \quad [\because t = x^3]$

Hence, $\frac{d}{dx}(\sin x^3) = 3x^2 \cos x^3$.

(ii) $y = \sin^3 x = (\sin x)^3$

Put $\sin x = t$, so that $y = t^3$ and $t = \sin x$

$\therefore \quad \frac{dy}{dt} = 3t^2$ and $\frac{dt}{dx} = \cos x$.

Hence, $\frac{dy}{dx} = \left(\frac{dy}{dt} \times \frac{dt}{dx}\right) = 3t^2 \cos x = 3\sin^2 x \cos x. \quad [\because t = \sin x]$

(iii) Let $y = e^{\sin x}$.

Put $\sin x = t$, so that $y = e^t$ and $t = \sin x$

$\therefore \quad \frac{dy}{dt} = e^t$ and $\frac{dt}{dx} = \cos x$.

Hence, $\frac{dy}{dx} = \left(\frac{dy}{dt} \times \frac{dt}{dx}\right) = e^t \cos x = e^{\sin x}.\cos x \quad [\because t = \sin x]$

Illustration 3

Question: **Differentiate : $\frac{e^x}{(1+\sin x)}$ w.r.t. x.**

Solution: Using the quotient rule, we have:

$$\frac{d}{dx}\left(\frac{e^x}{1+\sin x}\right) = \frac{(1+\sin x).\frac{d}{dx}(e^x) - e^x.\frac{d}{dx}(1+\sin x)}{(1+\sin x)^2}$$

$$= \frac{(1+\sin x).e^x - e^x(\cos x)}{(1+\sin x)^2} = \frac{(1+\sin x - \cos x)\ e^x}{(1+\sin x)^2}.$$

Illustration 4

Question: **If the distance s (in metres) travelled by a particle is given as a function of time t (in seconds) by the expression $s = t^{3/2}$, find its speed at $t = 25$ s.**

Solution:

Speed = Rate of change of distance *w.r.t.* time

$$= \frac{ds}{dt} = \frac{d}{dt}(t^{3/2}) = \frac{3}{2}t^{\frac{3}{2}-1} = \frac{3}{2}\sqrt{t}$$

Speed at $t = 25$ s. : $\left.\frac{ds}{dt}\right|_{t=25} = \frac{3}{2}\sqrt{25} = 7.5$ m/s.

2.3 MAXIMA AND MINIMA

Absolute maximum value of a function

A function $f(x)$ is said to have the greatest value or absolute maximum value at a point a in its domain, if $f(x) \le f(a)$ for all x in the domain of $f(x)$.

Absolute minimum value of a function

A function $f(x)$ is said to have the smallest value or absolute minimum value at a point a in its domain, if $f(a) \le f(x)$ for all x in the domain of $f(x)$.

Local maximum value of a function

We say that c is a point of local maximum of a function $f(x)$, if there is an open interval $\boldsymbol{I}$ containing c such that $f(x) < f(c)$ for all $x \in \boldsymbol{I}$.

Local minimum value of a function

We say that c is a point of local minimum of a function $f(x)$, if there is an open interval $\boldsymbol{I}$ containing c such that $f(c) < f(x)$ for all $x \in \boldsymbol{I}$.

Important formulae/points

To find maxima and minima

Working rule

(i) Find $f'(x)$.

(ii) Solve $f'(x) = 0$.
Each value of x so obtained is a candidate for maximum or minimum
Let $x = c$ be one of its points.

(iii) Find $f''(x)$ and put $x = c$ to get $f''(c)$.

Now, if $f''(c) < 0$, then $x = c$ is a point of local maximum;

if $f''(c) > 0$, then $x = c$ is a point of local minimum

if $f''(c) = 0$, then use other method to decide.

Illustration 5

Question: **Find all the points of local maxima and minima and the corresponding maximum and minimum values of the function** $\mathbf{f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105}$.

Solution:

$$f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$$

$\therefore\ f'(x) = -3x^3 - 24x^2 - 45x = -3x(x^2 + 8x + 15)$

Now, $f'(x) = 0 \Rightarrow -3x(x^2 + 8x + 15) = 0$

$\Rightarrow -3x(x + 5)(x + 3) = 0 \quad \Rightarrow x = 0$ or $x -5$ or $x = -3$.

Thus, $x = 0$; $x = -5$ and $x = -3$ are the candidates for local maxima or minima.

Moreover, $f''(x) = (-9x^2 - 48x - 45)$

Case I:

When $x = 0$. We have $f''(0) = -45 < 0$.

So, $x = 0$ is a point of local maximum.

And, local maximum value at $x = 0$ is $f(0) = 105$.

Case II:

When $x = -5$. We have $f''(-5) = -30 < 0$.

So, $x = -5$ is a point of local maximum.

And, local maximum value at $x = -5$ is $f(-5) = \frac{295}{4}$.

Case III:

When $x = -3$. We have $f''(-3) = 18 > 0$.

So, $x = -3$ is a point of local minima.

Local minimum value at $x = -3$ is $f(-3) = \frac{231}{4}$.

2.4 INTEGRAL CALCULUS

Here, we will be doing the reverse of Differential Calculus *e.g.*, determination of the distance travelled by a particle in a given time interval with its speed as a function of time specified.

If $v = \frac{ds}{dt} = 3t^2 - 4t + 1$

We write the same equation in integral calculus as

$$s = \int v\,dt = \int (3t^2 - 4t + 1)dt$$

where the symbol $\int$ is used to denote integral.

If we think backwards, we see that s can be taken as

$$s = t^3 - 2t^2 + t + \text{(any constant)}$$

to get $\frac{ds}{dt} = 3t^2 - 4t + 1$

We say that $t^3 - 2t^2 + t + c$, where c is an arbitrary constant, is the indefinite integral of $3t^2 - 4t + 1$ w.r.t. t, because of the presence of an arbitrary constant.

To find the distance travelled from time $t = 2$ s to $t = 10$ s, we take the difference of values of s at these time instants.

We write it as

$$s_{10} - s_2 = \int_2^{10} (3t^2 - 4t + 1)dt = (t^3 - 2t^2 + t + c)\Big|_2^{10}$$

$$= [(10)^3 - 2(10)^2 + (10) + c] - [(2)^3 - 2(2)^2 + (2) + c] = 810 - 2 = 808 \text{ metres.}$$

This process is called definite integration because the arbitrary constant vanishes.

Interestingly, this value represents the area enclosed by the speed-time graph, the time axis between t = 2s and t = 10 s.

We now state:

If the derivative of $F(x)$ w.r.t. x is $f(x)$, then the antiderivative, also called integration of $f(x)$ w.r.t. x is $F(x) + c$.

i.e., if $\frac{d}{dx}F(x) = f(x)$

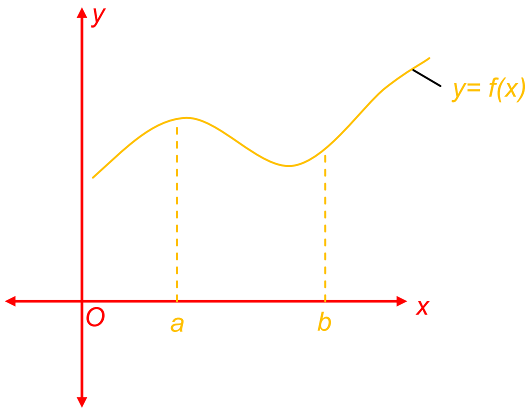
then $\int f(x)dx = F(x) + c$

where f(x) is called the integrand.

Definite integral of f(x) w.r.t. x is written as $\int_a^b f(x)dx = F(b) - F(a)$

where a is called the lower limit of integration and b is called the upper limit of integration.

$\int_a^b f(x)dx$ = Area bounded by the curve $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ (generally). (refer to the adjoining figure)



2.4.1 Formulae

On the basis of differentiation and the definition of integration, we have the following results:-

1. $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n,\ n \neq -1 \Rightarrow \int x^n dx = \frac{x^{n+1}}{(n+1)} + C$

2. $\frac{d}{dx}(\log|x|) = \frac{1}{x} \Rightarrow \int \frac{1}{x}dx = \log|x| + C$

3. $\frac{d}{dx}(e^x) = e^x \Rightarrow \int e^x dx = e^x + C$

4. $\frac{d}{dx}\left(\frac{a^x}{\log a}\right) = a^x \Rightarrow \int a^x dx = \frac{a^x}{\log a} + C$

5. $\frac{d}{dx}(\sin x) = \cos x \Rightarrow \int \cos x\, dx = \sin x + C$

6. $\frac{d}{dx}(-\cos x) = \sin x \Rightarrow \int \sin x\, dx = -\cos x + C$

7. $\frac{d}{dx}(\tan x) = \sec^2 x \Rightarrow \int \sec^2 x\, dx = \tan x + C$

8. $\frac{d}{dx}(-\cot x) = \operatorname{cosec}^2 x \Rightarrow \int \operatorname{cosec}^2 x\, dx = -\cot x + C$

9. $\frac{d}{dx}(\sec x) = \sec x \tan x \Rightarrow \int \sec x \tan x \, dx = \sec x + C$

10. $\frac{d}{dx}(-\text{cosec } x) = \text{cosec } x \cot x \Rightarrow \int \text{cosec } x \cot x \, dx = -\text{cosec} x + C$

11. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \Rightarrow \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$

12. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{(1+x^2)} \Rightarrow \int \frac{1}{(1+x^2)} dx = \tan^{-1} x + C$

13. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}} \Rightarrow \int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + C$

Important formulae/points

Some rules of integration

(i) $\int kf(x)dx = k\int f(x)dx$, where k is a constant.

(ii) $\int \{f(x)+g(x)\}dx = \int f(x)dx + \int g(x)\, dx$

(iii) $\int \{f(x)-g(x)\}dx = \int f(x)dx - \int g(x)dx$

Illustration 6

Question: **Evaluate : (i)** $\int (5x^3 + 2x^{-5} - 7x + \frac{1}{\sqrt{x}} + \frac{5}{x})dx$

(ii) $\int (3\sin x - 4\cos x + 5\sec^2 x - 2\text{cosec}^2 x)dx$.

Solution: We have:

(i) $\int (5x^3 + 2x^{-5} - 7x + \frac{1}{\sqrt{x}} + \frac{5}{x})dx$

$= 5\int x^3 dx + 2\int x^{-5}dx - 7\int x\, dx + \int x^{-1/2}dx + 5\int \frac{1}{x}dx$

$= 5.\frac{x^4}{4} + 2.\frac{x^{-4}}{(-4)} - 7.\frac{x^2}{2} + \frac{x^{1/2}}{(1/2)} + 5\log|x| + C$

$= 5.\frac{x^4}{4} - \frac{1}{2x^4} - \frac{7x^2}{2} + 2\sqrt{x} + 5\log|x| + C.$

(ii) $\int (3\sin x - 4\cos x + 5\sec^2 x - 2\text{cosec}^2 x)dx$

$= 3\int \sin x\, dx - 4\int \cos x\, dx + 5\int \sec^2 x\, dx - 2\int \text{cosec}^2 x\, dx$

$= (-3\cos x) - 4\sin x + 5\tan x - 2(-\cot x) + C$

$= -3\cos x - 4\sin x + 5\tan x + 2\cot x + C$

Illustration 7

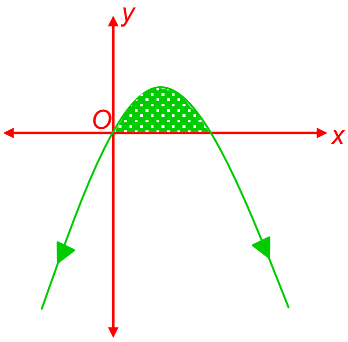
Question **Sketch the region bounded by the curve $y = 2x - x^2$ and the x-axis and find its area, using integration:-**

Solution: The given curve is, $y = 2x - x^2$. Some of the values of x and y satisfying the given equation are given below :

x	–1	0	1/2	1	3/2	2	3
y	–3	0	3/4	1	3/4	0	–3

Plot the points (–1, –3), (0, 0), $\left(\frac{1}{2}, \frac{3}{4}\right), (1, 1), \left(\frac{3}{2}, \frac{3}{4}\right), (2, 0)$ and (3, –3).

Join these points with a free hand to obtain a rough sketch of the graph of the given function. The required region is the shaded portion. (Note that it is a parabola opening downwards)



Clearly, the curve cuts x-axis, where $y = 0$.

i.e., $2x - 2x^2 = 0$ or $x(2 - x) = 0$

or $x = 0$ and $x = 2$

$\therefore$ Required area $= \int_0^2 y\,dx = \int_0^2 (2x - x^2)\,dx = \left[x^2 - \frac{x^3}{3}\right]_0^2 = \frac{4}{3}$ sq. units.

PROFICIENCY TEST–I

The Following Questions Deal With The Basic Concepts Of This Section. Answer The Following Briefly. Go To The Next Section Only If Your Score Is At Least 80%. Do Not Consult The Study Material While Attempting These Questions.

1. **Differentiate (a) xe^x (b) xsinx (c) x log x w.r.t. *X*.**

2. **If $y = \log \sin \frac{x}{2}$ then find $\frac{dy}{dx}$**

3. **Differentiate (a) $\frac{x}{\sin x}$ (b) $\frac{\log x}{x}$ w.r.t. *X*.**

4. **The mass of a body is 2.5 kg. It is in motion and its velocity *v* after *t* is $v = \frac{t^3}{3} + \frac{t^2}{2} + 1$. Calculate the force acting on the body at the time t = 3s.**

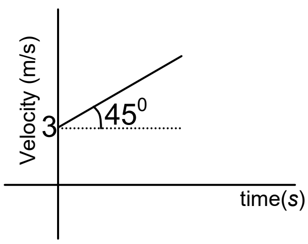
5. **If $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$, find $\frac{dy}{dx}$.**

6. **If velocity *v* depends on time *t* as $v = -t^2 + 3t + 1$ (where t is in second). At what time the velocity of particle is maximum?**

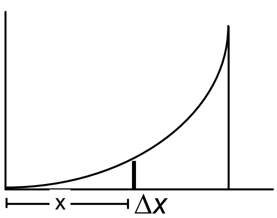
7. **The height reached in time *t* by a particle thrown upward with a speed *u* is given by $h = ut - \frac{1}{2}gt^2$, where *g* is a constant. Find the time taken in reaching the maximum height.**

8. **If force *f* varies with displacement *x* as $f = 3x^2 + 4$. The work done by force if particle moves from $x = 2$ m to $x = 4$ m is**

9. **If the velocity of particle varies with time as shown in figure. The average velocity of particle between $t = 0$ to $t = 4$s is**



10. **Figure shows the curve $y = x^2$. Find the area of the shaded part between $x = 0$ and $x = 6$.**



11. **Evaluate $\int_0^t A\sin(\omega t)\, dt$ where *a* and ω are constants.**

ANSWERS TO PROFICIENCY TEST–I

1. **(A)** $e^x(1+x)$ **(B)** $\sin x + x\cos x$ **(C)** $1+\log x$

2. $\frac{1}{2}\cot\frac{x}{2}$

3. **(A)** $\frac{\sin x - x\cos x}{\sin^2 x}$ **(B)** $\frac{1-\log x}{x^2}$

4. **30 N**

5. $\tan\frac{\theta}{2}$

6. $\frac{3}{2}$ **S**

7. $t = \frac{u}{g}$.

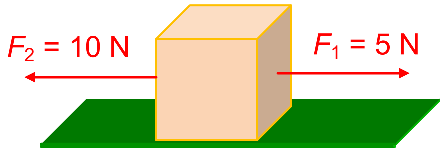
8. **64 J**

9. **5 M/S**

10. **72 SQUARE UNIT**

11. $\frac{A}{\omega}(1-\cos\omega t)$

Suppose a block of mass M is placed on a smooth horizontal surface. There are two forces F_1 and F_2 acting on the block as shown in the figure.

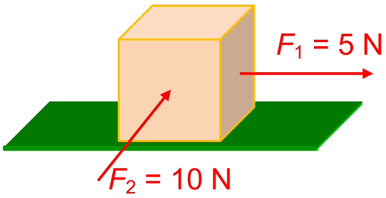


Now the question is in which direction will the block move? And what will be the net force on the block?

You can answer it easily. The block will move towards left and net force will be $(10 - 5) = 5$ N towards left.

Now think of the situation when these forces are neighter in the same direction nor opposite to each-other.

Suppose F_1 and F_2 are perpendicular to each other acting on the same block as shown in figure.

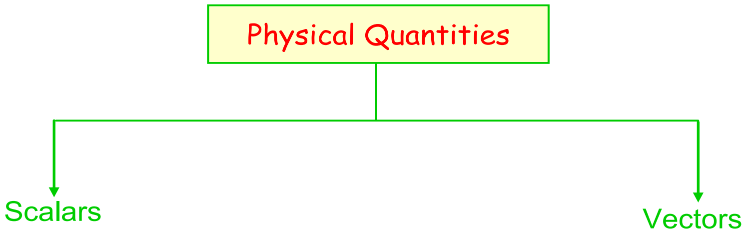


We repeat the same question. In which direction will the block move and what will be the net force? It will be difficult to answer. Why? Since you do not know about vectors.

Similar problems will be faced in other physical relations. We will now discuss vectors in detail.

4 SCALARS AND VECTORS

In the last lesson we have already discussed about physical quantities. All physical quantities have been categorised into two parts.



Scalars: Scalars are those physical quantities, which have only magnitude but no direction.

Examples: density, time, temperature, energy, mass, distance, speed etc.

Vectors: Vectors are those physical quantities, which have both magnitude and direction and obey the vector law of addition.

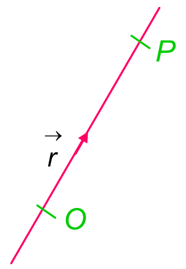
Examples: displacement, velocity, acceleration, force etc.

☞ A vector must obey the vector law of addition *otherwise it will not be a vector although having both magnitude as well as direction*.

Example: current has both magnitude and direction, but it is not a vector. It is a scalar quantity because it does not obey the vector law of addition, which we will learn in this lesson.

4.1 REPRESENTATION OF VECTOR

(i) Geometrical Method: Geometrically a vector is represented by the directed line segment i.e., by a line to which a direction has been assigned with an arrow-head in the direction of the vector and whose length is proportional to the magnitude of the vector.



To represent a vector geometrically, a line is drawn parallel to the direction of the vector and put an arrow on the line along the direction of the vector. Now this directed line segment, namely, *OP* as shown in figure represents the vector in magnitude and direction. It is written as $\vec{OP}$. '*O*' is called the 'initial point' of the vector and *P*, the 'terminal point' of it. The vector $\vec{OP}$ is also written as $\vec{r}$ i.e., we also write $\vec{r} = \vec{OP}$.

To represent a physical quantity in a vector form, we put an arrow above the symbol of the physical quantity. For example, velocity is denoted by *v* but in vector form it is represented as $\vec{v}$ which is read as velocity vector.

Magnitude of vector is called absolute value indicated by $|\vec{v}|$ (modulus of velocity vector)

(ii) Analytical Method: In this method vector is represented in terms of unit vector ($\hat{i}$, $\hat{j}$ and $\hat{k}$), which we will see in details later on.

4.2 DIFFERENT TYPES OF VECTOR

(i) Like Vectors: Two parallel vectors having the same sense of direction are called like vectors and opposite sense of direction are called **unlike vectors**.

Example: Let body *A* is moving toward east and another body *B* is also moving in the same direction, then these two velocity vectors are called **like vectors**.

(ii) Collinear/Parallel Vectors: Vectors having the same line of action or having lines of action parallel to the same line are called **collinear vectors**.

They may have the same sense or opposite sense of direction.

(iii) Coplanar Vectors: Vectors are said to be coplanar if they lie in the same plane or they are parallel to the same plane, otherwise they are said to be **non-coplanar vectors**.

(iv) Zero Vector/Null Vector: Vectors having zero magnitude are called **zero or null vectors**. It is denoted by $\vec{O}$.

In case of a zero vector, its initial and the final point coincide and its direction is indeterminate.

In this case initial and terminal points coincide.

$\vec{OO}$, $\vec{AA}$ are zero vectors

Examples: The velocity vector of a stationary particle, the acceleration vector of an object moving with uniform velocity are zero vector.

(v) Unit Vector: A unit vector is a vector of unit magnitude and points in a particular direction. It is used to specify the direction only. Unit vector is represented by putting a cap (^) over the quantity unit vector is unit less and dimension less.

The unit vector in the direction of $\vec{F}$ is denoted by $\hat{F}$ and defined by

$$\hat{F} = \frac{\vec{F}}{|\vec{F}|}, \quad \Rightarrow \quad \vec{F} = |\vec{F}|\,\hat{F} \qquad \ldots (1)$$

To represent any value of force in this direction, we can use this unit vector. Like λ is magnitude of force and we multiply λ with $\hat{F}$, it will give force along the direction of *F*.

$$\vec{F} = \lambda \hat{F}$$

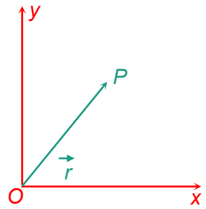
(vi) Negative of a Vector: The vector whose magnitude is same as that of $\vec{a}$ but the direction is opposite to that of $\vec{a}$ is called the negative of $\vec{a}$ and is written as $-\vec{a}$.

(vii) Position Vectors: Position vector represents the position of an object in a plane with respect to a fixed-point that is origin of a coordinate system.

Let *O* be the origin and *P* be any point then $\vec{OP}$ is called the position vector of *P* with respect to the origin *O*.

It can be represented by a single letter $\vec{r}$.

$\vec{OP} = \vec{r}$. The length of the vector $\vec{r}$ represents the magnitude of vector and its direction is the direction in which *P* lies as seen from *O*.



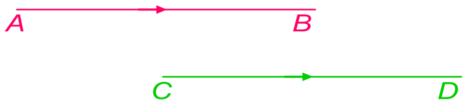
(viii) Equal Vectors: Two vectors are said to be equal if

(i) their magnitudes are equal

(ii) they are parallel

(iii) they have the same sense of direction. They needn't have the same initial point.

In the figure shown length of *AB* equal to length of *CD* and *AB* and *CD* have same sense of direction.



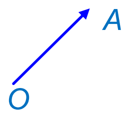
$\therefore \quad \vec{AB} = \vec{CD}$

So here $\vec{AB}$ & $\vec{CD}$ are two equal vectors.

4.3 PROPERTIES OF VECTORS

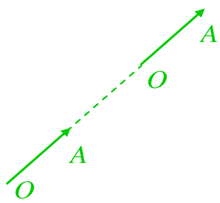
(i) Vectors can be bound: The vectors in which point of application and direction both are fixed.

Such type of vector's position is fixed and it is called bound vectors.



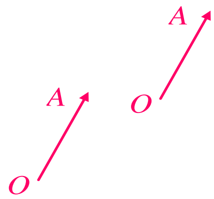
(ii) Vectors can be sliding: In sliding the point of application is shifted along the original line of action without any change in magnitude and direction.

Shifted position of $\vec{OA}$ is shown in the figure.



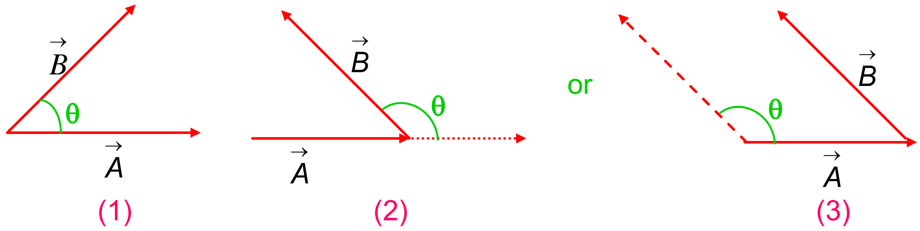
(iii) Vectors can be moved freely: In free movement of vector its point of application can be changed without any change in magnitude and direction of the vector and is always parallel to the original line of action.

As $\vec{OA}$ has been shifted in a new position as shown in figure.



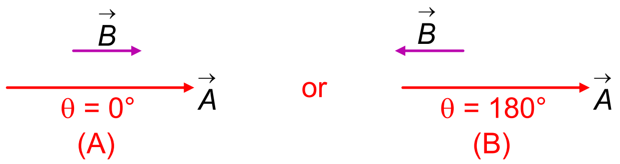
☞ ***It is worth noting that in this lesson all vectors has been used as free vector.***

(iv) Angle between two vectors means smaller of the two angles between the vectors when they are placed tail to tail by displacing either of the vectors parallel to itself (i.e., $0 \le \theta \le \pi$).



Here θ represents the angle between $\vec{A}$ and $\vec{B}$.

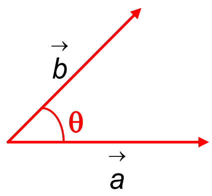
(v) Angle between collinear vectors is always zero or 180°.



5 ADDITION OF VECTORS

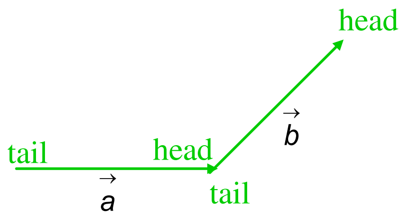
5.1 GRAPHICAL METHOD

(i) Triangle Law of Vector Addition: Let us consider two vectors $\vec{a}$ and $\vec{b}$ as shown.



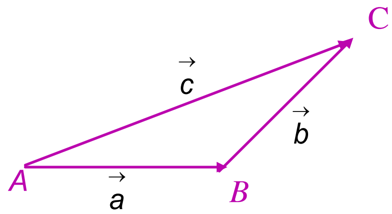
Now to get the sum of these two vectors ($\vec{a} + \vec{b}$), shift any two vectors parallel to itself until the tail of one vector is at the head of another vector (using the sliding and free vector properties).

Here we place the vector $\vec{b}$ in such a way that its tail touch the head of vector $\vec{a}$, which is shown in figure.



Let $\vec{AB} = \vec{a}$ and $\vec{BC} = \vec{b}$

Then the line joining the tail of $\vec{a}$ and head of vector $\vec{b}$, $\vec{AC}$ gives the sum of vector $\vec{a}$ and $\vec{b}$



Let $\quad \vec{AC} = \vec{c}$

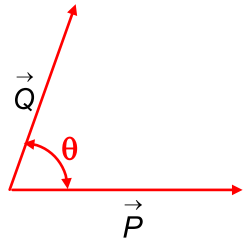
$$\vec{a} + \vec{b} = \vec{c} \quad \Rightarrow \quad \vec{AB} + \vec{BC} = \vec{AC}$$

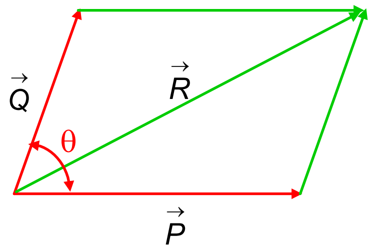
This is known as triangle law of vector addition.

We can define in this way. ***"If two vectors are represented by the two sides of a triangle taken in order, then their resultant or vector sum is represented by the third side of the triangle taken in opposite order".***

(ii) Parallelogram law of vector addition: Let there are two vectors $\vec{P}$ and $\vec{Q}$ in such a way that they have common initial point and different direction as shown in figure.

Let us complete a parallelogram with $\vec{P}$ and $\vec{Q}$ as its adjacent sides.

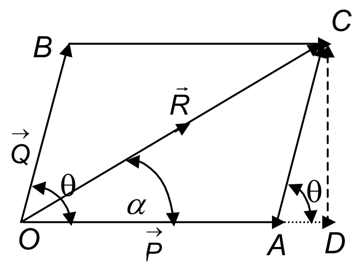




Then, the diagonal of parallelogram passing through their common point will represent in magnitude and direction the resultant of the vectors $\vec{P}$ and $\vec{Q}$.

$$\vec{R} = \vec{P} + \vec{Q}$$

Magnitude of vector $\vec{R}$ is given by the length of diagonal of the parallelogram which can be calculated in this way



Let the two vectors $\vec{P}$ and $\vec{Q}$ be represented in magnitude and direction by $\overrightarrow{OA}$ and $\overrightarrow{OB}$ respectively. Considering *OA* and *OB* as two adjacent sides, parallelogram *OACB* is constructed.

We drop a perpendicular *CD* on *OA* produced.

In right angled triangle *ACD*.

$$\cos\theta = \frac{AD}{AC} \Rightarrow AD = \text{AC}\cos\theta$$

or, $AD = Q\cos\theta$

Also, $CD = Q\sin\theta$

Now in right angle triangle *ODC*

$$OC^2 = OD^2 + DC^2$$

$$\left|\vec{R}\right|^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$

$$= P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \qquad \ldots (2)$$

Let α be the angle, which the resultant $\vec{R}$ makes with $\vec{P}$.

$$\tan\alpha = \frac{DC}{OD} = \frac{Q\sin\theta}{P + Q\cos\theta}, \quad \alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right) \qquad \ldots (3)$$

Some special case

Case I:

When $\vec{P}$ and $\vec{Q}$ are in same direction, it means $\theta = 0°$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 0°}$$

$$R = \sqrt{(P+Q)^2} = P + Q$$

In this case resultant is maximum

$$\tan\alpha = \frac{Q\sin 0°}{P + Q\cos 0°} \Rightarrow \alpha = 0$$

Case II:

When $\vec{P}$ and $\vec{Q}$ are perpendicular to each other, it means θ = 90°

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90^\circ}$$

$$R = \sqrt{P^2 + Q^2}$$

$$\tan\alpha = \frac{Q}{P} \Rightarrow \alpha = \tan^{-1}\left(\frac{Q}{P}\right)$$

Case III:

When $\vec{P}$ and $\vec{Q}$ are in opposite direction, it means θ = π

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\pi} = \sqrt{P^2 + Q^2 - 2PQ}$$

$$= \sqrt{(P-Q)^2}$$

$$= (P - Q)$$

In this case resultant will be minimum.

$$\tan\alpha = \frac{Q\sin\pi}{P + Q\cos\pi} = 0 \Rightarrow \alpha = 0$$

Case IV:

When two vectors are of same magnitude $(|\vec{P}| = |\vec{Q}|)$

$$R = \sqrt{P^2 + P^2 + 2P^2\cos\theta}$$

$$= \sqrt{2P^2(1+\cos\theta)}$$

$$= \sqrt{2P^2(2\cos^2\frac{\theta}{2})}$$

$$= 2P\cos\frac{\theta}{2}$$

$$\tan\alpha = \frac{P\sin\theta}{P + P\cos\theta} = \frac{\sin\theta}{2\cos^2\frac{\theta}{2}} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$$

$$\tan\alpha = \tan\frac{\theta}{2}$$

$$\Rightarrow \alpha = \theta/2$$

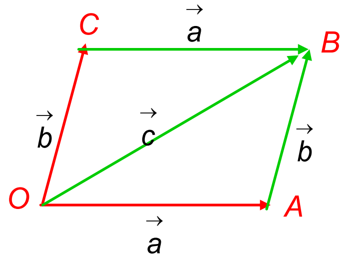
☞ *If two vectors are of equal magnitude then the resultant of vectors bisects the angle between them.*

5.2 PROPERTIES OF VECTOR ADDITION

(i) It Obeys Commutative law

If $\vec{a}$ and $\vec{b}$ are any two vectors,

then $\vec{a} + \vec{b} = \vec{b} + \vec{a}$



Proof: Let $\vec{OA} = \vec{CB} = \vec{a}$

$$\vec{AB} = \vec{OC} = \vec{b}$$

$$\vec{OB} = \vec{C}$$

In ΔOAB, $\vec{OA} + \vec{AB} = \vec{OB}$ (from triangle law of vector addition).

$$\Rightarrow \quad \vec{a} + \vec{b} = \vec{c} \qquad \ldots \text{(i)}$$

In ΔOCB, $\vec{OC} + \vec{CB} = \vec{OB}$

$$\Rightarrow \quad \vec{b} + \vec{a} = \vec{c} \qquad \ldots \text{(ii)}$$

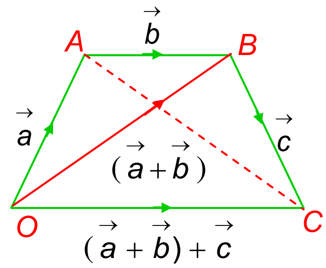
from equation (i) & (ii)

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

(ii) It obeys associative law

If $\vec{a}$, $\vec{b}$, and $\vec{c}$ are any three vectors then

$$\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$$



Proof : Let $\vec{OA} = \vec{a}$

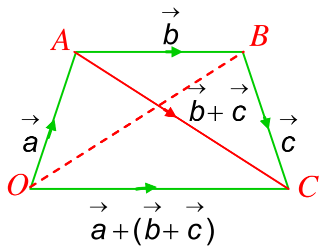
$\vec{AB} = b$, $\vec{BC} = \vec{c}$

In ΔOAB, $\vec{OB} = \vec{OA} + \vec{AB} = \vec{a} + \vec{b}$

In ΔOBC, $\vec{OC} = \vec{OB} + \vec{BC} = (\vec{a} + \vec{b}) + \vec{c}$ $\qquad \ldots$(i)

In ΔABC $\vec{AC} = \vec{AB} + \vec{BC} = \vec{b} + \vec{c}$

In ΔOAC, $\vec{OC} = \vec{OA} + \vec{AC} = \vec{a} + (\vec{b} + \vec{c})$ $\quad \ldots$ (ii)



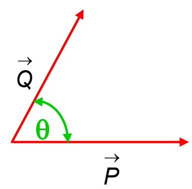
from equation (i) and (ii)

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

6 SUBTRACTION OF VECTORS

Subtraction of vector can be defined in terms of addition of two vectors.

$$\vec{P} - \vec{Q} = \vec{P} + (-\vec{Q})$$



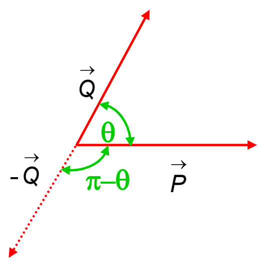
Let $\vec{P}$ and $\vec{Q}$ are at an angle θ as shown in the figure.

To get $(\vec{P} - \vec{Q})$, first we will draw a vector $-\vec{Q}$ as shown below.

Then angle between $\vec{P}$ and $-\vec{Q}$ will be (π - θ).

$$|\vec{P} + (-\vec{Q})| = \sqrt{P^2 + Q^2 + 2PQ\cos(\pi - \theta)}$$

$$|\vec{P} - \vec{Q}| = \sqrt{P^2 + Q^2 - 2PQ\cos\theta} \qquad \ldots (4)$$



☞ ***Subtraction is not commutative**, i.e.,*

$$\vec{P} - \vec{Q} \neq \vec{Q} - \vec{P}$$

Illustration 8

Question: **There are two vectors having magnitude 3 units and 4 units respectively**

(a) What should be the resultant if angle between them is 60°

(b) What should be the angle between them if the magnitude of resultant is (i) 1 unit (ii) 5 units?

Solution:

(a) $|\vec{a}| = 3$ units, $|\vec{b}| = 4$ units and $\theta = 60°$

$$\vec{R} = \vec{a} + \vec{b}$$

$$R = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

$$= \sqrt{9 + 16 + 2.3.4\cos 60°} = \sqrt{25 + 12} = \sqrt{\mathbf{37}}\ \textbf{units}$$

(b) (i) $|R| = \sqrt{a^2 + b^2 + 2.3.4\cos\theta}$

$$1 = \sqrt{9 + 16 + 2.3.4\cos\theta}$$

$$1 = 25 + 24\cos\theta$$

$$\cos\theta = \frac{-24}{24} = -1 = \cos\pi \Rightarrow \theta = \pi$$

(ii) $(5)^2 = \sqrt{25 + 24\cos\theta}$

$$25 = 25 + 24\cos\theta$$

$$0 = 24\cos\theta \Rightarrow \cos\theta = 0$$

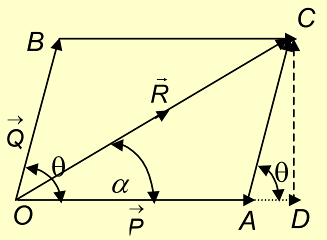
$\therefore$ **$\theta = \pi/2$**

Important formulae/points

- **Vectors**: Physical quantities having both magnitude and direction
- Triangle law of vector addition $\vec{R} = \vec{P} + \vec{Q}$

  $|R| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$,

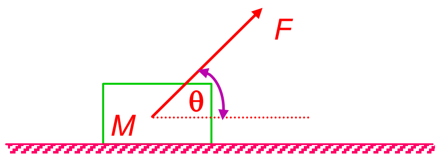
  $\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$
- $|\vec{P} - \vec{Q}| = \sqrt{P^2 + Q^2 - 2PQ\cos\theta}$



7 RESOLUTION OF VECTORS

It is the process of splitting a single vector into two or more vectors in different directions which together produce the same effect as is produced by the single vector alone. The vectors into which the given single vector is splitted are called **component of vectors.** In fact, the resolution of a vector is just opposite to composition of vectors.

Let there is force acting on a block, which is on a frictionless surface, at an angle θ with the horizontal as shown in figure.



Now due to this force it will move towards right and go some distance.

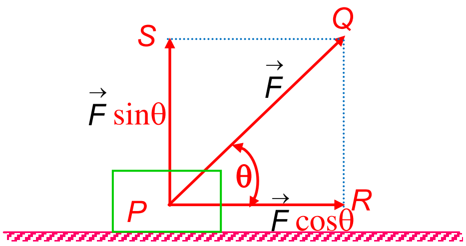
We want to know the force, which is cause of rightward motion.

It can be known by the components of the force *F*. The cause of horizontal motion is horizontal component which will be *F* cos θ.

We can understand these things in this way

Let force F is acting from P to Q

We want to know the force in the direction PR and PS.



In right angle ΔPQR

$$\cos\theta = \frac{PR}{PQ} = \frac{F_{PR}}{F_{PQ}}$$

$\Rightarrow \quad PR = PQ\cos\theta$

$\therefore \quad \vec{F}_{PR} = \vec{F}\cos\theta$

So, ***horizontal component*** $= \boldsymbol{F\cos\theta}$

In right angle ΔPQR

$$\sin\theta = \frac{\vec{RQ}}{\vec{PQ}} = \frac{\vec{F}_{RQ}}{\vec{F}_{PQ}}$$

$$\vec{F}_{RQ} = \vec{F}\sin\theta$$

$$\vec{F}_{PS} = \vec{F}_{RQ} = \vec{F}_{PQ}\sin\theta$$

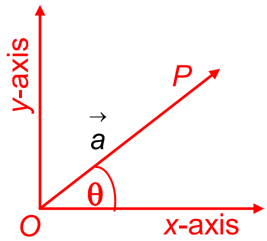
So, *vertical component = F sin θ*

In this way we resolve the vector in two perpendicular directions.

☞ ***We can remember it in this way, towards θ the component will be a factor of cos θ and other component perpendicular to it will be a factor of sin θ.***

7.1 RECTANGULAR COMPONENTS OF A VECTOR IN A PLANE

When a vector is resolved along the two axes of a rectangular co-ordinate system (i.e., x and y axis), the components of the vector are called rectangular components of a vector.



Let there is a vector $\vec{a}$ from O to P at an angle θ from the x-axis.

Magnitude of component of $\vec{a}$ along x-axis $(a_x) = a\cos\theta$

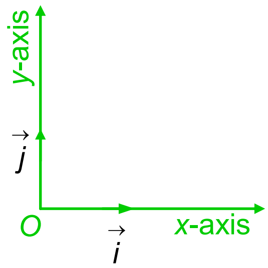
Magnitude of component of $\vec{a}$ along y-axis $(a_y) = a\sin\theta$

These two components are called rectangular components of the vector $\vec{a}$.

Representation of Rectangular components

These components are represented in terms of unit vector.

Unit vector along x, and y-*axis* are represented by $\hat{i}$ and $\hat{j}$ respectively as shown in figure.



$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$

these unit vectors are perpendicular to each other.

Rectangular components of vector $\vec{a}$ in terms of unit vector can be written as

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

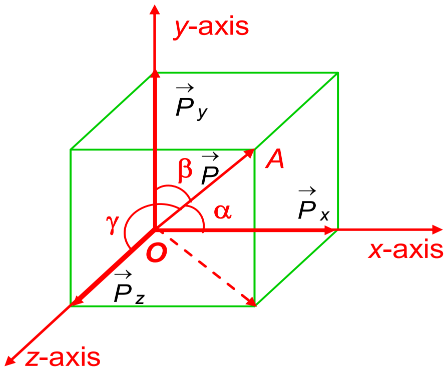
$$\vec{a} = a\cos\theta\,\hat{i} + a\sin\theta\,\hat{j}$$

7.2 RECTANGULAR COMPONENTS OF A VECTOR IN THREE DIMENSIONS

Components, which we get on resolving a vector, lie in space along three mutually perpendicular directions (i.e., x, y and z axes) are called rectangular components or orthogonal components.

The vector is called non-coplanar (three-dimensional) vector.

Let there be a non-coplanar vector $\vec{P}$ from O to A as shown in figure. Taking O as origin and a rectangular parallelopiped with its three edges along the three rectangular axes i.e., x, y and z axes, is constructed.



Here $\vec{P}$ represent the diagonal of the parallelopiped whose intercepts along these axes are $\vec{P}_x, \vec{P}_y$ and $\vec{P}_z$ respectively which are three orthogonal components of $\vec{P}$.

$$P^2 = P_x^2 + P_y^2 + P_z^2$$

or, $$P = \sqrt{P_x^2 + P_y^2 + P_z^2} \qquad \ldots \text{(i)}$$

Let α, β and γ are the angles between $\vec{P}$ and x, y and z-axis, respectively, then we can get the components in this way

$$\cos\alpha = \frac{P_x}{P} \Rightarrow P_x = P\cos\alpha$$

$$\cos\beta = \frac{P_y}{P} \Rightarrow P_y = P\cos\beta$$

$$\cos\gamma = \frac{P_z}{P} \Rightarrow P_z = P\cos\gamma$$

Here $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called the direction cosines of the vector $\vec{P}$.

Putting the values of P_x, P_y and P_z in (i), we get

$$P^2 = P^2\cos^2\alpha + P^2\cos^2\beta + P^2\cos^2\gamma$$

or, $$P^2 = P^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$$

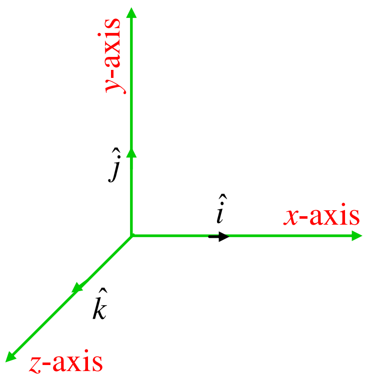
or, $$(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 1$$

It means that the sum of the squares of the direction cosines of a vector is always unity.

Representation of Rectangular Components or Orthogonal Components

These components are represented in terms of unit vector.

Unit vectors along x, y, and z-axis are represented by $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively as shown in figure.



$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$

these unit vectors are perpendicular to each other.

Orthogonal components in terms of unit vector can be written as

$$\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$$

$$\vec{P} = P\cos\alpha\,\hat{i} + P\cos\beta\,\hat{j} + P\cos\gamma\,\hat{k}$$

$$|\vec{P}| = \sqrt{(P_x)^2 + (P_y)^2 + (P_z)^2} \quad \ldots (5)$$

Illustration 9

Question: **A mass of 2 kg lies on a plane making an angle 30° to the horizontal. Resolve its weight along and perpendicular to the plane. Assume g = 10 m/s^2.**

Solution: As shown in the figure, the component of weight along the plane = $mg\sin\theta$

$= 2 \times 10 \times \sin 30 = 10$ N.

The component of weight perpendicular to plane

$= mg\cos 30°$

$= 2 \times 10 \times \sqrt{3}/2 =$ **17.3 N.**

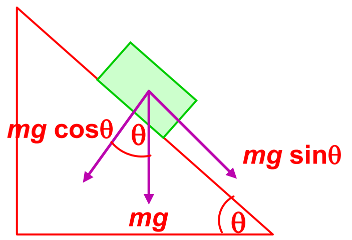


Illustration 10

Question: **If $\vec{P} = 2\hat{i} + 4\hat{j} - 5\hat{k}$, find (i) $|\vec{P}|$ and (ii) the direction cosines of the vector $\vec{P}$.**

Solution:

(i) $P = \sqrt{P_x^2 + P_y^2 + P_z^2}$

$= \sqrt{(2)^2 + (4)^2 + (-5)^2} = \sqrt{45}$

(ii) $\cos\alpha = \dfrac{P_x}{P} = \dfrac{2}{\sqrt{45}}$

$\cos\beta = \dfrac{P_y}{P} = \dfrac{4}{\sqrt{45}}$

$\cos\gamma = \dfrac{P_z}{P} = \dfrac{-5}{\sqrt{45}}$

Important formulae/points

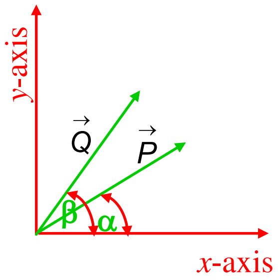
- **Orthogonal components in terms of unit vector**

  $\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k} = P\cos\alpha\hat{i} + P\cos\beta\hat{j} + P\cos\gamma\hat{k}$

- $|\vec{P}| = \sqrt{(P_x)^2 + (P_y)^2 + (P_z)^2}$

In this method, vectors are added when it is represented in terms of unit vector. At first coplanar vectors are resolved in two components and non-coplanar vectors are resolved in three components, which are perpendicular to each other, and all components in one direction are added together.

Let there are two co-planer vectors $\vec{P}$ and $\vec{Q}$ making angles α and β respectively with x-axis as shown in the figure.



$$\vec{P} = P_x\hat{i} + P_y\hat{j}$$
$$= P\cos\alpha\hat{i} + P\sin\alpha\hat{j}$$
$$\vec{Q} = Q_x\hat{i} + Q_y\hat{j}$$
$$= Q\cos\beta\hat{i} + Q\sin\beta\hat{j}$$
$$\vec{P}+\vec{Q} = (P_x\hat{i} + P_y\hat{j}) + (Q_x\hat{i} + Q_y\hat{j})$$
$$= (P_x + Q_x)\hat{i} + (P_y + Q_y)\hat{j}$$
$$|\vec{P}+\vec{Q}| = \sqrt{(P_x + Q_x)^2 + (P_y + Q_y)^2}$$

Let $\vec{R}$ be their sum

$$\therefore \quad \vec{R} = (P_x + Q_x)\hat{i} + (P_y + Q_y)\hat{j}$$
$$\vec{R} = R_x\hat{i} + R_y\hat{j} \text{ where}$$
$$R_x = (P_x + Q_x) \text{ and } R_y = (P_y + Q_y)$$

☞ ***This method can be used in addition and subtraction for any number of vectors.***

Illustration 11

Question: **If $\vec{P} = 2\hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{Q} = 4\hat{i} - 2\hat{j} + \hat{k}$, then calculate the magnitude of**

(a) $\vec{P} + \vec{Q}$ (b) $\vec{P} - \vec{Q}$ (c) $2\vec{P} + \vec{Q}$

***Solution*:**

(a) $\vec{P}+\vec{Q} = (2\hat{i} + 3\hat{j} - 2\hat{k}) + (4\hat{i} - 2\hat{j} + \hat{k})$

$= 6\hat{i} + \hat{j} - \hat{k}$

Magnitude = $\sqrt{36+1+1} = \sqrt{38}$

(b) $\vec{P}-\vec{Q} = (2\hat{i} + 3\hat{j} - 2\hat{k}) - (4\hat{i} - 2\hat{j} + \hat{k})$

$= 2\hat{i} + 3\hat{j} - 2\hat{k} - 4\hat{i} + 2\hat{j} - \hat{k}$

$= -2\hat{i} + 5\hat{j} - 3\hat{k}$

Magnitude = $\sqrt{4+25+9} = \sqrt{38}$

(c) $2\vec{P}+\vec{Q} = (4\hat{i} + 6\hat{j} - 4\hat{k}) + (4\hat{i} - 2\hat{j} + \hat{k})$

$= 8\hat{i} + 4\hat{j} - 3\hat{k}$

magnitude = $\sqrt{64+16+9} = \sqrt{89}$

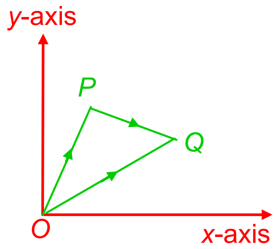
Illustration 12

Question: **If the position vectors of *P* and *Q* be respectively ($\hat{i} + 3\hat{j} - 7\hat{k}$) and ($5\hat{i} - 2\hat{j} + 4\hat{k}$), find $\vec{PQ}$**

Solution: Let O be the origin

Given $\vec{OP} = \hat{i} + 3\hat{j} - 7\hat{k}$

$\vec{OQ} = 5\hat{i} - 2\hat{j} + 4\hat{k}$



By triangle law of vector addition,

$\vec{OP} + \vec{PQ} = \vec{OQ}$

$\vec{PQ} = \vec{OQ} - \vec{OP}$

$= (5\hat{i} - 2\hat{j} + 4\hat{k}) - (\hat{i} + 3\hat{j} - 7\hat{k})$

$= \mathbf{(4\hat{i} - 5\hat{j} + 11\hat{k})}$

Important formulae/points

- **Analytical method of vector addition**
  $R_x = (P_x + Q_x)$, $R_y = (P_y + Q_y)$

9 MULTIPLICATION OF VECTORS

9.1 MULTIPLICATION OF VECTOR BY SCALAR

When any vector is multiplied by a scalar quantity like with any real number, it is just multiplied like an algebraic product.

Example: Let there be a vector $\vec{V}$ and a scalar quantity λ. After multiplication product will be $\vec{V}' = \lambda\vec{V}$

☞ *If λ is + ve, then $\vec{V}'$ and $\vec{V}$ will have same direction.*

☞ *If λ is –ve then $\vec{V}'$ and $\vec{V}$ will have opposite direction.*

9.2 MULTIPLICATION OF VECTORS

(i) Scalar Product (Dot Product)

(ii) Vector Product (Cross Product)

(i) Scalar Product (Dot Product): Scalar product of two vectors is defined as the product of the magnitude of two vectors with cosine of smaller angle between them.

It is always a scalar, so it is called as scalar product.

Thus if $\vec{A}$ and $\vec{B}$ are two vectors having angle θ between them, then their scalar (Dot) product written as $\vec{A}.\vec{B}$ and read as $\vec{A}$ dot $\vec{B}$ is defined as

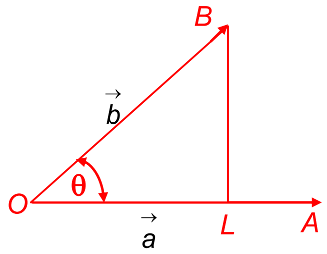
$$\vec{A}.\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta \qquad \text{.... (6)}$$

Geometrical Meaning

Let $\vec{OA} = \vec{a}$ and $OB = \vec{b}$ as shown in figure and

$\angle AOB = \theta$

From B, we drop a perpendicular BL on OA.



$$\cos\theta = \frac{OL}{OB}$$

$$OL = OB\cos\theta$$

By definition

$$\vec{a}.\vec{b} = ab\cos\theta$$

$$= (OA).(OB)\cos\theta$$

$$= (OA).(OL)$$

$\vec{a}.\vec{b}$ = (Mod of $\vec{a}$) (Projection of $\vec{b}$ on $\vec{a}$)

Properties

(i) It is always a scalar and it will be positive if angle between them is acute, negative if angle between them is obtuse and zero if angle between them is 90°

(ii) It obeys commutative law $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

(iii) It obeys distributive law $\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B}+\vec{A}.\vec{C}$

(iv) By definition $\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

$$\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}|\,|\vec{B}|}$$

$$\theta = \cos^{-1}\left(\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}\right)$$

where θ is angle between two vectors.

(v) Scalar (Dot) product of two mutually perpendicular vectors is zero i.e.,

$$(\vec{A}.\vec{B}) = AB\cos 90° = 0$$

(vi) Scalar (Dot) product will be maximum when $\theta = 0°$ i.e.,

vectors are parallel to each other. $(\vec{A}.\vec{B})_{max} = |A|\,|B|$

(vii) It $\vec{a}$ and $\vec{b}$ are unit vectors then $|\vec{a}| = |\vec{b}| = 1$ and $\vec{a}.\vec{b} = 1.1\cos\theta = \cos\theta$

(viii) Dot product of unit vectors $\hat{i}, \hat{j}, \hat{k}$

$$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$$

$$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$$

(ix) Square of a vector $\vec{a}.\vec{a} = |a||a|\cos 0^0 = a^2$

(x) If the two vectors $\vec{A}$ and $\vec{B}$, in terms of their rectangular components, are

$\vec{A} = A_x\,\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}$, then,

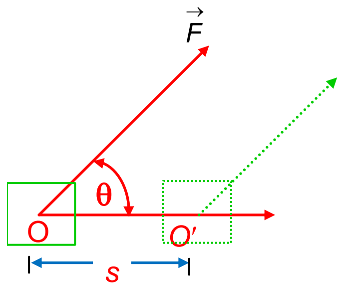
$$\vec{A}.\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\,\hat{k}).\,(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Example of Dot Product

Work: Work is the dot product of force and displacement vector.

Let $\vec{F}$ be the force acting on block at angle θ with the horizontal and the block is displaced by this force from O to O'.



Let $\quad \vec{OO'} = \vec{S}$

Work $= \vec{F}.\vec{S}$

$= |\vec{F}|\,|\vec{S}|\cos\theta$

Illustration 13

***Question*:** **Find the angle between two vectors $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{B} = \hat{i} - \hat{k}$.**

Solution: $A = |\vec{A}| = \sqrt{(2)^2 + (1)^2\,(-1)^2} = \sqrt{6}$

$B = |\vec{B}| = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$

$\vec{A}.\vec{B} = (2\hat{i} + \hat{j} - \hat{k}).\,(\hat{i} - \hat{k}) = (2)\,(1) + (-1)\,(-1) = 3$

Now, $\cos\theta = \dfrac{\vec{A}.\vec{B}}{AB} = \dfrac{3}{\sqrt{6}.\sqrt{2}} = \dfrac{3}{\sqrt{12}} = \dfrac{\sqrt{3}}{2}$

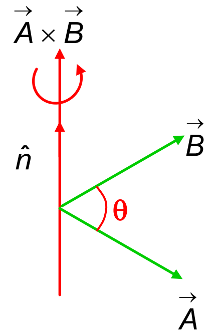
$\therefore \quad \theta = 30°$

Important formulae/points

- **Scalar product** $\vec{A}.\vec{B} = |A|\,|B|\cos\theta$
- $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$ **and** $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

(ii) Vector Product/Cross Product: *Vector Product of two vectors is defined as a vector having magnitude equal to product of the magnitude of two vectors with sine of smaller angle between them,* *and direction perpendicular to the plane containing the two vectors and in the sense of advancement of a right handed screw rotated from first vector to the second vector through smaller angle between them.*

If $\vec{A}$ and $\vec{B}$ are two vectors, then their vector product is written as $\vec{A}\times\vec{B}$ is vector and is read as $\vec{A}$ cross $\vec{B}$. It is defined as

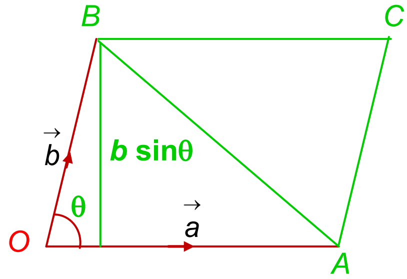


$$\vec{A}\times\vec{B} = |\vec{A}||\vec{B}| \sin\theta\ \hat{n} \qquad \ldots (7)$$

where $\hat{n}$ is unit vector along the direction of $\vec{A}\times\vec{B}$

Geometrical meaning

Let $OACB$ be a parallelogram having side $\vec{OA} = \vec{a}$ and $\vec{OB} = \vec{b}$ and $\angle AOB = \theta$.



Area of parallelogram $OACB$

$= 2$ area of triangle OAB

$$= 2\left(\frac{ab\sin\theta}{2}\right)$$

$$= ab\sin\theta = |\vec{a}\times\vec{b}|$$

Thus, $\vec{a}\times\vec{b}$ is a vector whose modulus is the area of the parallelogram formed by the two vectors as the adjacent sides and direction is perpendicular to both $\vec{a}$ and $\vec{b}$.

Properties

(i) Cross product of two vectors is not commutative

$$\vec{a}\times\vec{b} \neq \vec{b}\times\vec{a}$$

$$\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$$

(ii) cross product is not associative

$$\vec{a}\times(\vec{b}\times\vec{c}) \neq (\vec{a}\times\vec{b})\times\vec{c}$$

(iii) cross product obey distributive law

$$\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b} + \vec{a}\times\vec{c}$$

(iv) If $\theta = 0$ or π it means two vectors are collinear.

$$\vec{a}\times\vec{b} = 0$$

and conversely, if $\vec{a}\times\vec{b} = 0$ then the vector $\vec{a}$ and $\vec{b}$ are parallel provided $\vec{a}$ and $\vec{b}$ are non-zero vectors.

☞ *This may be regarded as a test to decide whether the given two vectors are parallel or not.*

(v) If $\theta = 90°$, then

$$\vec{a}\times\vec{b} = |a||b|\sin 90° = |a||b|\,\hat{n}$$

(vi) The vector product of any vector with itself is $\vec{0}$

$$\vec{a} \times \vec{a} = \vec{0}$$

(vii) If $\vec{a} \times \vec{b} = 0$, then

$$\vec{a} = 0 \text{ or } \vec{b} = 0 \text{ or } \vec{a} \parallel \vec{b}$$

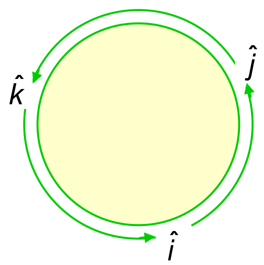
(viii) If $\vec{a}$ and $\vec{b}$ are unit vectors, then $\vec{a} \times \vec{b} = 1.1 \sin\theta\, \hat{n} = \sin\theta\, \hat{n}$

(ix) Cross product of unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$
$$\hat{i} \times \hat{j} = k = -\hat{j} \times \hat{i}$$
$$\hat{j} \times \hat{k} = \hat{i} = -\hat{k} \times \hat{j}$$
$$\hat{k} \times \hat{i} = \hat{j} = -\hat{i} \times \hat{k}$$

These result can be remembered easily with the help of following method

Around a circle placed $\hat{i}, \hat{j}$ and $\hat{k}$ in anticlockwise direction as shown in figure.



Now cross product of any two unit vectors will give third unit vectors and it will be +ve if on this circle given two vectors are in anticlockwise direction otherwise it will be –ve

$$\hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{i} = -\hat{k}$$
$$\hat{j} \times \hat{k} = \hat{i}, \quad \hat{k} \times \hat{j} = -\hat{i}$$
$$\hat{k} \times \hat{i} = \hat{j}, \quad \hat{i} \times \hat{k} = -\hat{j}$$

(x) If the two vectors $\vec{A}$ and $\vec{B}$ in terms of their rectangular component are

$$\vec{A} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$$

$$\vec{B} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$$

$$\vec{A} \times \vec{B} = (a_1\hat{i} + b_1\hat{j} + c_1\hat{k}) \times (a_2\hat{i} + b_2\hat{j} + c_2\hat{k})$$

It can be found by the determinant method

$$\text{i.e., } \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \hat{i}(b_1c_2 - b_2c_1) - \hat{j}(a_1c_2 - a_2c_1) + \hat{k}(a_1b_2 - a_2b_1)$$

Application and example

Torque: Torque about a point is the cross product of vector joining the point to the point of application of force and the force.

$$\text{Torque } (\tau) = \vec{r} \times \vec{F}$$

When $\vec{r}$ is vector joining the point to the point of application of force and $\vec{F}$ is force vector.

Illustration 14

Question: **Find a unit vector perpendicular to $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{B} = \hat{i} - \hat{j} + \hat{k}$ both**

Solution: As we have read,

$\vec{C} = \vec{A} \times \vec{B}$ is a vector perpendicular to both $\vec{A}$ and $\vec{B}$. Hence, a unit vector $\hat{n}$ perpendicular to $\vec{A}$ and $\vec{B}$ can be written as

$$\hat{n} = \frac{\vec{C}}{C} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

Here, $$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(3+1) + \hat{j}(1-2) + \hat{k}(-2-3) = 4\hat{i} - \hat{j} - 5\hat{k}$$

Further, $|\vec{A} \times \vec{B}| = \sqrt{(4)^2 + (-1)^2 + (-5)^2} = \sqrt{42}$

$\therefore$ The desired unit vector is

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} \quad \text{or,} \quad \hat{n} = \frac{1}{\sqrt{42}}(4\hat{i} - \hat{j} - 5\hat{k})$$

Important formulae/points

- $\vec{A} \times \vec{B} = |A||B|\sin\theta\hat{n}$
  $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$ and $\hat{i} \times \hat{j} = \hat{k} = -\hat{j} \times \hat{i}$, $\hat{j} \times \hat{k} = \hat{i} = -\hat{k} \times \hat{j}$, $\hat{k} \times \hat{i} = \hat{j} = -\hat{i} \times \hat{k}$

PROFICIENCY TEST–II

The Following Questions Deal With The Basic Concepts Of This Section. Answer The Following Briefly. Go To The Next Section Only If Your Score Is At Least 80%. Do Not Consult The Study Material While Attempting These Questions.

1. **The magnitude of vectors *a*, *b* and *c* are respectively 12, 5 and 13 units and** $\vec{A}+\vec{B}=\vec{C}$**. What is the angle between *a* and *b* ?**

2. **What is the property of two vectors** $\vec{A}$ **and** $\vec{B}$**, if**

   (a) $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$ **(b)** $\vec{A}+\vec{B}=\vec{A}-\vec{B}$ **?**

3. **Two vectors each of magnitude 5 units have an angle 60° between them. Find the magnitude of**
   (a) the sum of the vectors
   (b) the difference of the vectors

4. **Two forces 8 n and 10 n are acting upon a body. What will be the maximum and minimum resultant force on the body ?**

5. **Two forces of 5 n and 10 n are acting with an inclination of 120° between them. What is the angle which their resultant makes with 10 n ?**

6. **One of the rectangular components of a velocity of 100 kmh**$^{-1}$ **is 50 km h**$^{-1}$**. Find the other component**

7. **An aeroplane takes off at an angle of 30**^{0} **to the horizontal. If the component of its velocity along the horizontal is 250 km h**$^{-1}$**, what is its actual velocity? Also find the vertical component of its velocity.**

8. **Find the direction cosines of** $5\hat{i}+2\hat{j}+4\hat{k}$**.**

9. **Resolve horizontally and vertically a force *f* = 8 n which makes an angle of 45° with the horizontal.**

10. **Resolve a weight of 10 n acting horizontally in two directions which are parallel and perpendicular to a slope inclined at 30° to the horizontal.**

11. **Given:** $\vec{a}=3\hat{i}+4\hat{j}$ **and** $\vec{b}=3\hat{j}+4\hat{k}$**. Calculate the magnitude** $\vec{a}+\vec{b}$**.**

12. **The maximum and minimum numerical value of the resultant of two forces respectively 16 n and 4n, then calculate the numerical value of individual forces.**

13. **If** $\vec{P}=2\hat{i}+3\hat{j}-\hat{k}$ **and** $\vec{Q}=-\hat{i}-5\hat{j}+2\hat{k}$**, find the angle between** $\vec{P}$ **and** $\vec{Q}$**.**

14. **Prove that the vectors** $\vec{A}=2\hat{i}-3\hat{j}+\hat{k}$ **and** $\vec{B}=\hat{i}+\hat{j}+\hat{k}$ **are mutually perpendicular.**

15. $\hat{i}$ and $\hat{j}$ are unit vectors along x and y-axis respectively. What is the magnitude and direction of the vectors $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$? What are the components of a vector $\vec{A}=2\hat{i}+3\hat{j}$ along the direction $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$?

16. A particle moves from position $\vec{r}_1=3\hat{i}+2\hat{j}-6\hat{k}$ to position $\vec{r}_2=14\hat{i}+13\hat{j}-9\hat{k}$ under the action of a force $(4\hat{i}+\hat{j}+3\hat{k})$ newton. Calculate the work done.

17. For what value of *m*, the vector $\vec{A}=2i+3\hat{j}-6\hat{k}$ is perpendicular to $\vec{B}=3\hat{i}-m\hat{j}+6\hat{k}$?

18. If $\overrightarrow{a} = 2\hat{i} + 3\hat{j} + \hat{k}$ and *b* = $\hat{i} + 2\hat{j} + 3\hat{k}$ find $\vec{a}\times\vec{b}$

19. Let a force $\overrightarrow{F}$ be acting on a body free to rotate about a point *o* and let $\overrightarrow{r}$ the position vector of any point *p* on the line of action of the force. Then torque $(\overrightarrow{\tau})$ of this force about point *o* is defined as $\overrightarrow{\tau} = \overrightarrow{r}\times\overrightarrow{F}$. Given, $\overrightarrow{F}=(2\hat{i}+3\hat{j}-\hat{k})$ n and $\overrightarrow{r} = (\hat{i} - \hat{j}+6\hat{k})$m.
find the torque of this force.

20. Determine a unit vector which is perpendicular to both $\vec{P}=2\hat{i}-\hat{j}-\hat{k}$ and $\vec{Q}=\hat{i}+\hat{j}-2\hat{k}$.

ANSWERS TO PROFICIENCY TEST–I

1. **$\pi/2$**

2. **(a) the vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other (b) $\vec{B}$ is a null vector.**

3. **(a) $5\sqrt{3}$ units (b) 5 units**

4. **(i) 18 n (ii) 2 n**

5. **$\beta = 30°$**

6. $50\sqrt{3}$ kmh^{-1}

7. 288.68 km h^{-1}, 144.34 km h^{-1}

8. $\frac{5}{\sqrt{45}}, \frac{2}{\sqrt{45}}, \frac{4}{\sqrt{45}}$

9. $F_x = F_y = 4\sqrt{2}$ **n**

10. $5\sqrt{3}$ **n and 5 n**

11. **8.602 units**

12. **10 n, 6 n**

13. $\theta = \cos^{-1}\left(-\frac{19}{2\sqrt{105}}\right)$

15. $\frac{5}{\sqrt{2}}$ **and** $-\frac{1}{\sqrt{2}}$ **units**

16. **46 j**

17. **– 10**

18. $7\hat{i} - 5\hat{j} + \hat{k}$

19. $\vec{\tau} = (-17\hat{i} + 13\hat{j} + 5\hat{k})$ **n-m**

20. $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

SOLVED OBJECTIVE EXAMPLES

Example 1:

The 10th term from the end in the AP 5, 8, 11, …… is 95 then the number of terms in the *AP* are

(a) 38 (b) 40 (c) 42 (d) 43

Solution:

If the number of terms in the A.P. are n, then 10th term from the end will be $(n-9)th$ term from starting.

$\therefore \quad 5+(n-10)\times 3 = 95 \Rightarrow \quad n = 40$

$\therefore$ **(b)**

Example 2:

$\sin 20° \sin 70° - \cos 20° \cos 70° =$

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

Solution:

$\sin 20° \sin 70° - \cos 20° \cos 70° = -\cos(70° + 20°) = 0$

$\therefore$ **(b)**

Example 3:

What is the unit vector perpendicular to the following vectors $2\hat{i}+2\hat{j}-\hat{k}$ and $6\hat{i}-3\hat{j}+2\hat{k}$

(a) $\frac{\hat{i}+10\hat{j}-18\hat{k}}{5\sqrt{17}}$ (b) $\frac{\hat{i}+10\hat{j}+18\hat{k}}{5\sqrt{17}}$ (c) $\frac{\hat{i}-10\hat{j}-18\hat{k}}{5\sqrt{17}}$ (d) $\frac{\hat{i}+10\hat{j}+18\hat{k}}{5\sqrt{17}}$

Solution:

$$\hat{n} = \frac{\vec{A}\times\vec{B}}{(\vec{A}\times\vec{B})}$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -1 \\ 6 & -3 & 2 \end{vmatrix} = \hat{i}(4-3) - \hat{j}(4+6) + \hat{k}(-6-12) = i - 10\hat{j} - 18\hat{k}$$

$$|\vec{A}\times\vec{B}| = 5\sqrt{17}$$

$$\hat{n} = \frac{\hat{i}-10\hat{j}-18\hat{k}}{5\sqrt{17}}$$

$\therefore$ **(c)**

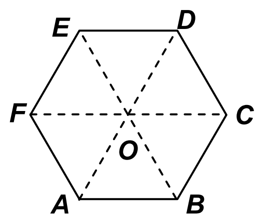
Example 4:

Figure shows *ABCDEF* as a regular hexagon. What is the value of

$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}$

(a) $\overrightarrow{AO}$ (b) $2\overrightarrow{AO}$

(c) $4\overrightarrow{AO}$ (d) $6\overrightarrow{AO}$



Solution:

$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}$

$= \overrightarrow{AB}+(\overrightarrow{AB}+\overrightarrow{BC})+(\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD})+(\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE})+(\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}+\overrightarrow{EF})$

$\because \quad \overrightarrow{AB} = -\overrightarrow{DE}$, $\overrightarrow{BC} = -\overrightarrow{EF}$, $\overrightarrow{CD} = -\overrightarrow{FA}$

$\therefore \quad \overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF} = 5\overrightarrow{AB}+4\overrightarrow{BC}+3\overrightarrow{CD}+2\overrightarrow{DE}+\overrightarrow{EF}$

$= 3\overrightarrow{AB}+3\overrightarrow{BC}+3\overrightarrow{CD} = 3\overrightarrow{AD} = 6\overrightarrow{AO}$

$\therefore$ **(d)**

Example 5:

The sum of the magnitudes of two forces acting at point is 18 and the magnitude of their resultant is 12. If the resultant is at 90° with the force of smaller magnitude, what are the, magnitudes of forces

(a) 12, 5 **(b) 14, 4** **(c) 5, 13** **(d) 10, 8**

Solution:

Let magnitude of force $\vec{A}$ is smaller than magnitude of force $\vec{B}$

$$\frac{A+B=18}{\sqrt{A^2+B^2+2AB\cos\theta}}=12$$

$$\tan 90^\circ=\frac{B\sin\theta}{A+\cos\theta}\Rightarrow A+B\cos\theta=0 \quad\Rightarrow\quad \cos\theta=-\frac{A}{B}$$

$$\sqrt{A^2+B^2+2AB\left(-\frac{A}{B}\right)}=12$$

$$\sqrt{A^2+B^2-2A^2}=12 \quad\Rightarrow\quad \sqrt{B^2-A^2}=12$$

We get, $A=5,\ B=13$

$\therefore$ **(c)**

Example 6:

Value of $I=\int_0^1 (2x+1)^2\,dx$ **is**

(a) 26 **(b) 13** **(c)** $\frac{13}{2}$ **(d)** $\frac{13}{3}$

Solution:

$$I=\int_0^1 (2x+1)^2 dx=\frac{1}{6}\left[(2x+1)^3\right]_0^1$$

$$=\frac{1}{6}[27-1]=\frac{26}{6}=\frac{13}{3}$$

$\therefore$ **(d)**

Example 7:

If $y=\sin^3(3x^3),\ \frac{dy}{dx}$ **will be**

(a) $\cos^3(3x)^3$ **(b)** $\sin^3(9x^2)$

(c) $27x^2\sin^2(3x^3)\cos(3x^3)$ **(d)** $3\sin^2(3x^3)\cos(3x^3)$

Solution:

$$y=\sin^3(3x^3)$$

$$\frac{dy}{dx}=\frac{d\{\sin^3(3x^3)\}}{\sin(3x^3)}\times\frac{d\{\sin(3x^3)\}}{d(3x^3)}\times\frac{d\{3x^3\}}{dx}$$

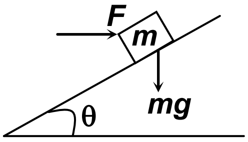
$$=3\sin^2(3x^3)\times\cos\ 3x^3\times 9x^2$$

$$=27x^2\sin^2(3x^3)\cos(3x^3)$$

$\therefore$ **(c)**

Example **8:**

The forces acting on a block placed on a smooth inclined plane of angle θ is as shown. Net force acting on the block down the incline is



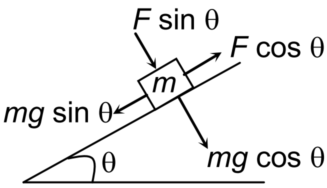
(a) $mg\sin\theta + F\cos\theta$ (b) $mg\sin\theta - F\cos\theta$ (c) $mg\cos\theta - F\sin\theta$ (d) $mg\cos\theta + F\sin\theta$

Solution:

Net force down the incline

$= mg\sin\theta - F\cos\theta$

∴ **(b)**

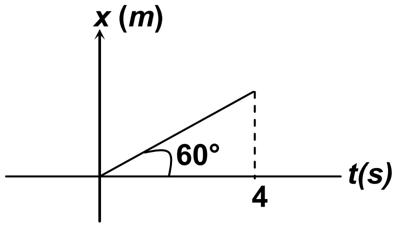


Example **9:**

Position time curve of a particle moving along x-axis is as shown in x-t curve. The position of particle at *t* = 4s will be,

(a) $4\sqrt{3}m$ (b) $2\sqrt{3}m$

(c) $2\,m$ (d) $1\,m$



Solution:

$$\tan 60° = \frac{x}{t}$$

$$x = 4 \times \sqrt{3}\ m$$

∴ **(a)**

Example **10:**

Let $\vec{A} = A\cos\theta\hat{i} + A\sin\theta\hat{j}$ be any vector another vector $\vec{B}$ which is perpendicular to $\vec{A}$ can be expressed as

(a) $B\cos\theta\hat{i} - B\sin\theta\hat{j}$ (b) $B\sin\theta\hat{i} - B\cos\theta\hat{j}$

(c) $B\cos\theta\hat{i} + B\sin\theta\hat{j}$ (d) $B\sin\theta\hat{i} + B\cos\theta\hat{j}$

Solution:

Let $\vec{B} = B_x\hat{i} + B_j\hat{j} + B_z\hat{k}$ is perpendicular to $\vec{A}$

∴ $\vec{A}\ .\ \vec{B} = 0$

⇒ $A_xB_x + A_yB_y + A_zB_z = 0$

∴ **(b)**

SOLVED SUBJECTIVE EXAMPLES

***Example* 1:**

What is the torque of the force $\vec{F} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ N acting at the point $\vec{r} = 3\hat{i} + 2\hat{j} + 3\hat{k}$ m about the origin?

Solution:

Torque $\vec{\tau} = \vec{r} \times \vec{F}$

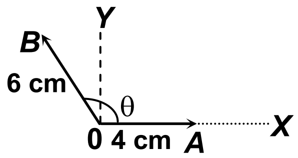
or, $\vec{\tau} = (3\hat{i} + 2\hat{j} + 3\hat{k}) \times (2\hat{i} - 3\hat{j} + 4\hat{k})$

$= 0 - 9\hat{k} - 12\hat{j} - 4\hat{k} + 0 + 8\hat{i} + 6\hat{j} + 9\hat{i} + 0$

$= 17\hat{i} - 6\hat{j} - 13\hat{k}$

***Example* 2:**

The resultant of vectors $\vec{OA}$ and $\vec{OB}$ is perpendicular to $\vec{OA}$. Find the angle *AOB*.



Solution:

Take the dotted lines as *X*, *Y* axes.

x-component of $\vec{OA}$ = 4 m, x-component of $\vec{OB}$ = 6 m $\cos\theta$

x-component of the resultant = $(4 + 6\cos\theta)m$

But it is given that the resultant is along Y-axis. Thus, $4 + 6\cos\theta = 0$ or, $\cos\theta = -\frac{2}{3}$.

***Example* 3:**

Write the unit vector in the direction of $\vec{A} = 5\hat{i} + \hat{j} - 2\hat{k}$.

Solution:

$|\vec{A}| = \sqrt{5^2 + 1^2 + (-2)^2} = \sqrt{30}$

The required unit vector is $\frac{\vec{A}}{|\vec{A}|}$

$= \frac{5}{\sqrt{30}}\hat{i} + \frac{1}{\sqrt{30}}\hat{j} - \frac{2}{\sqrt{30}}\hat{k}$.

***Example* 4:**

If $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = 4\hat{i} + 3\hat{j} + 2\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.

Solution:

We have $\vec{a}\,.\,\vec{b} = ab\cos\theta$

or, $\cos\theta = \frac{\vec{a}\,.\,\vec{b}}{ab}$

where θ is the angle between $\vec{a}$ and $\vec{b}$.

Now, $\vec{a}\,.\,\vec{b} = a_x b_x + a_y b_y + a_z b_z$

$= 2 \times 4 + 3 \times 3 + 4 \times 2 = 25$

Also, $a = \sqrt{a_x^2 + a_y^2 + a_z^2}$

$= \sqrt{4 + 9 + 16} = \sqrt{29}$

And $b = \sqrt{b_x^2 + b_y^2 + b_z^2} = \sqrt{16 + 9 + 4} = \sqrt{29}$

Thus, $\cos\theta = \frac{25}{29}$

or, $\theta = \cos^{-1}\left(\frac{25}{29}\right)$

Example 5:

If $\vec{A} = 2\hat{i} - 3\hat{j} + 7\hat{k}$, $\vec{B} = \hat{i} + 2\hat{k}$ and $\vec{C} = \hat{j} - \hat{k}$, find $\vec{A}.(\vec{B} \times \vec{C})$.

Solution:

$$\vec{B} \times \vec{C} = (\hat{i} + 2\hat{k}) \times (\hat{j} - \hat{k})$$
$$= \hat{i} \times (\hat{j} - \hat{k}) + 2\hat{k} \times (\hat{j} - \hat{k})$$
$$= \hat{i} \times \hat{j} - \hat{i} \times \hat{k} + 2\hat{k} \times \hat{j} - 2\hat{k} \times \hat{k}$$
$$= \hat{k} + \hat{j} - 2\hat{i} - 0 = -2\hat{i} + \hat{j} + \hat{k}$$
$$\vec{A}.(\vec{B} \times \vec{C}) = (2\hat{i} - 3\hat{j} + 7\hat{k}).(-2\hat{i} + \hat{j} + \hat{k})$$
$$= (2)(-2) + (-3)(1) + (7)(1) = 0$$

Example 6:

The volume of sphere is given by $V = \frac{4}{3}\pi R^3$ where *R* is the radius of the sphere. (a) Find the rate of charge of volume with respect to *R*. (b) Find the change in volume of the sphere as the radius is increased from 20.0 cm to 20.1 cm. Assume that the rate does not appreciably change between *R* = 20.0 cm to *R* = 20.1 cm.

Solution:

(a) $V = \frac{4}{3}\pi R^3$

or, $\frac{dV}{dR} = \frac{4}{3}\pi \frac{d}{dr}(R)^3 = \frac{4}{3}\pi.3R^2 = 4\pi R^2$.

(b) At *R* = 20 cm, the rate of change of volume with the radius is

$$\frac{dV}{dR} = 4\pi R^2 = 4\pi(400 \text{ cm}^2)$$
$$= 1600\ \pi\ \text{cm}^2$$

The change in volume as the radius changes from 20.0 cm to 20.1 cm is

$$\Delta V = \frac{dV}{dR}\Delta R$$
$$= (1600\ \pi\ \text{cm}^2)\ (0.1\ \text{m}) = 160\ \pi\ \text{cm}^3.$$

Example 7:

Find the derivative of the following functions with respect to *x*. (a) $y = x^2 \sin x$, (b) $y = \frac{\sin x}{x}$ and (c) $y = \sin(x^2)$.

Solution:

(a) $y = x^2 \sin x$

$$\frac{dy}{dx} = x^2 \frac{d}{dx}(\sin x) + (\sin x)\frac{d}{dx}(x^2)$$
$$= x^2 \cos x + (\sin x)(2x)$$
$$= x(2\sin x + x\cos x).$$

(b) $y = \frac{\sin x}{x}$

$$\frac{dy}{dx} = \frac{x\frac{d}{dx}(\sin x) - \sin x\left(\frac{dx}{dx}\right)}{x^2}$$
$$= \frac{x\cos x - \sin x}{x^2}$$

(c) $\frac{dy}{dx} = \frac{d}{dx^2}(\sin x^2).\frac{d(x)^2}{dx}$

$= \cos x^2(2x) = 2x\cos x^2$.

***Example* 8:**

Find the maximum or minimum values of the function $y = x + \frac{1}{x}$ for $x > 0$.

Solution:

$$y = x + \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}(x^{-1})$$

$$= 1(-x^{-2}) = 1 - \frac{1}{x^2}$$

For y to be maximum or minimum,

$$\frac{dy}{dx} = 0$$

or, $1 - \frac{1}{x^2} = 0$

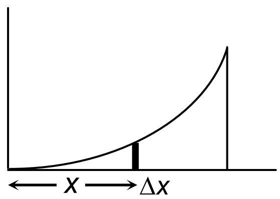
Thus, $x = 1$ or -1.

For $x > 0$ the only possible maximum or minimum is at $x = 1$. At $x = 1, y = x + \frac{1}{x} = 2$.

Near $x = 0$, $y = x + \frac{1}{x}$ is very large because of the term $\frac{1}{x}$. For very large x, again y is very large because of the term x. Thus, $x = 1$ must correspond to a minimum. Thus, y has only a minimum for $x > 0$. This minimum occurs at $x = 1$ and the minimum value of y is $y = 2$.

***Example* 9:**

Figure shows the curve $y = x^2$. Find the area of the shaded part between $x = 0$ and $x = 6$.



Solution:

The area can be divided into strips by drawing ordinates between $x = 0$ and $x = 6$ at a regular interval of dx. Consider the strip between the ordinates at x and $x + dx$. The height of this strip is $y = x^2$. The area of this strip is $dA = ydx = x^2dx$.

The total area of the shaded part is obtained by summing up these strip-areas with x varying from 0 to 6. Thus,

$$A = \int_0^6 x^2dx = \left[\frac{x^3}{3}\right]_0^6 = \frac{216-0}{3} = 72$$

***Example* 10:**

Evaluate $\int_0^1 A\sin\omega t dt$ where A and ω are constants.

Solution:

$$\int_0^t A\sin\omega t dt = A\left[\frac{-\cos\omega t}{\omega}\right]_0^t = \frac{A}{\omega}(1-\cos\omega t).$$

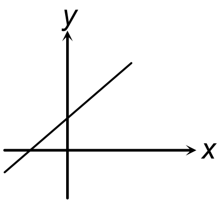
EXERCISE – I

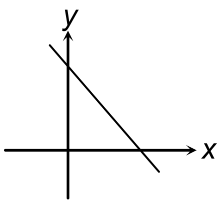
NEET SINGLE CHOICE CORRECT

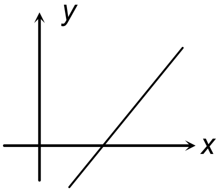
1. The value of sin (–150°) is

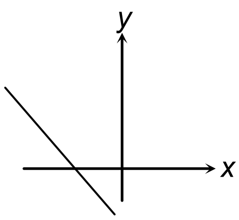
(a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

2. Which of the following may represent the curve $x = 2y - 3$

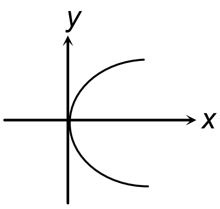
(a) 

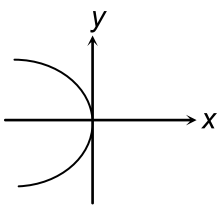
(b) 

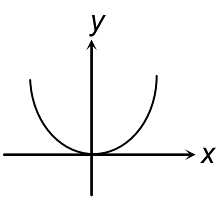
(c) 

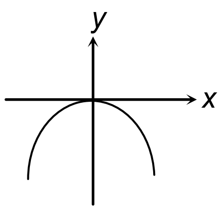
(d) 

3. Which of the following can represent the curve $x^2 = -2y$?

(a) 

(b) 

(c) 

(d) 

4. If $y = \frac{1}{2}\sin x^2, \frac{dy}{dx}$ will be,

(a) $\frac{1}{2}\cos x^2$ (b) $x\cos x^2$ (c) $\frac{1}{2}x^2\cos x^2$ (d) $\sin x$

5. Find out value of $I = \int_{-\pi/2}^{\pi/2} \sin 2x \; dx$

(a) zero (b) – 1 (c) 1 (d) 2

6. Velocity of a particle is given as $v = 2t^2 - 3$ m/s. The acceleration of particle at t = 3s will be

(a) 18 ms^{-2} (b) 12 ms^{-2} (c) 15 ms^{-2} (d) zero

7. If $\vec{A} = \hat{i} + \hat{j}$ and $\vec{B} = \hat{i} - \hat{k}$, the angle between $\vec{A}$ and $\vec{B}$ is

(a) zero (b) 180° (c) 60° (d) 90°

8. If $\vec{A} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{B} = 4\hat{j} + p\hat{i} + \hat{k}$ for what value of p, $\vec{A}$ and $\vec{B}$ will be collinear?

(a) 3 (b) – 3

(c) $-\frac{16}{3}$ (d) $\vec{A}$ and $\vec{B}$ cannot be collinear

9. Two sides of a triangle is represented by $\vec{a} = 3\hat{j}$ and $\vec{b} = 2\hat{i} - \hat{k}$. The area of triangle is

(a) 5 (b) $3\sqrt{5}$ (c) $\frac{3}{2}\sqrt{5}$ (d) $\sqrt{5}$

10. The value of the determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is

(a) $a+b+c$ (b) 0

(c) 1 (d) *abc*

11. The only integral root of the equation $\begin{vmatrix} 2-y & 2 & 3 \\ 2 & 5-y & 6 \\ 3 & 4 & 10-y \end{vmatrix} = 0$ is

(a) $y = 1$ (b) $y = 2$

(c) $y = 3$ (d) none of these

12. If *a*, 4, *b* are in A.P. and *a*, 2, *b* are in G.P., then *a*, 1, *b* are in

(a) H.P. (b) G.P.

(c) A.P. (d) none of these

13. The non-negative real root of the quadratic equation $3x^2 - 5x - 2 = 0$ is

(a) 3 (b) $\frac{1}{3}$ (c) 2 (d) $\frac{1}{2}$

14. The discriminant of the quadratic equation $ax^2 - 4ax + 2a + 1 = 0$ is

(a) $4a(2a+1)$ (b) $2a(2a+1)$ (c) $4a(2a-1)$ (d) $2a(4a-1)$

15. Two adjacent sides of a parallelogram are represented by the two vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$. What is the area of parallelogram

(a) 8 (b) $8\sqrt{3}$ (c) $3\sqrt{8}$ (d) 192

EXERCISE – II

IIT-JEE SINGLE CHOICE CORRECT

1. The sum of the first 11 terms of the *AP* 2, 6, 10, ……. is
(a) 238 (b) 242 (c) 246 (d) 254

2. If $\sin\theta = \frac{1}{3}$ then $2\operatorname{cosec}^2\theta + \cot^2\theta - 1 =$
(a) 15 (b) 25 (c) 50 (d) 45

3. If two vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $-4\hat{i} - 6\hat{j} - \lambda\hat{k}$ are parallel to each other then value of λ be
(a) 0 (b) – 2 (c) 3 (d) 4

4. Two vectors $\vec{A}$ and $\vec{B}$ are at right angles to each other, when
(a) $\vec{A} + \vec{B} = 0$ (b) $\vec{A} - \vec{B} = 0$ (c) $\vec{A} \times \vec{B} = 0$ (d) $\vec{A}.\vec{B} = 0$

5. the position vectors of points *A*, *B*, *C* and *D* are $A = 3\hat{i} + 4\hat{j} + 5\hat{k}, B = 4\hat{i} + 5\hat{j} + 6\hat{k}, C = 7\hat{i} + 9\hat{j} + 3\hat{k}$ and $D = 4\hat{i} + 6\hat{j}$ then the displacement vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are
(a) Perpendicular (b) Parallel
(c) Antiparallel (d) Inclined at an angle of 60°

6. If $\left|\vec{A} \times \vec{B}\right| = \left|\vec{A}.\vec{B}\right|$, then angle between $\vec{A}$ and $\vec{B}$ will be
(a) 30° (b) 45° (c) 60° (d) 90°

7. If $\vec{A}$ and $\vec{B}$ are perpendicular vectors and vector $\vec{A} = 5\vec{i} + 7\vec{j} - 3\vec{k}$ and $\vec{B} = 2\hat{i} + 2\hat{j} - a\hat{k}$. The value of *a* is
(a) – 2 (b) 8 (c) – 7 (d) – 8

8. If a vector $\vec{P}$ making angles α, β, and γ respectively with the *X*, *Y* and *Z* axes respectively. Then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$
(a) 0 (b) 1 (c) 2 (d) 3

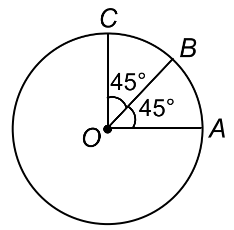
9. If the resultant of *n* forces of different magnitudes acting at a point is zero, then the minimum value of *n* is
(a) 1 (b) 2 (c) 3 (d) 4

10. Can the resultant of 2 vectors be zero
(a) Yes, when the 2 vectors are same in magnitude and direction
(b) No
(c) Yes, when the 2 vectors are same in magnitude but opposite in sense
(d) Yes, when the 2 vectors are same in magnitude making an angle of $\frac{2\pi}{3}$ with each other

11. A vector $\vec{a}$ is turned without a change in its length through a small angle $d\theta$. The value of $|\Delta\vec{a}|$ and Δa are respectively

(a) 0, $a\,d\theta$ (b) $a\,d\theta$, 0 (c) 0, 0 (d) None of these

12. Find the resultant of three vectors $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ shown in the following figure. Radius of the circle is R.



(a) $2R$ (b) $R(1+\sqrt{2})$

(c) $R\sqrt{2}$ (d) $R(\sqrt{2}-1)$

13. A force $\vec{F} = -K(y\hat{i} + x\hat{j})$ (where K is a positive constant) acts on a particle moving in the x-y plane. Starting from the origin, the particle is taken along the positive x-axis to the point $(a, 0)$ and then parallel to the y-axis to the point (a, a). The total work done by the force $\vec{F}$ on the particle is

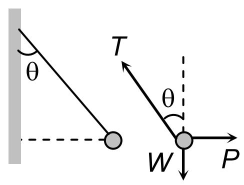
(a) $-2Ka^2$ (b) $2Ka^2$ (c) $-Ka^2$ (d) Ka^2

14. The vectors from origin to the points A and B are $\vec{A} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j} - 2\hat{k}$ respectively. The area of the triangle OAB be

(a) $\frac{5}{2}\sqrt{17}$ sq. unit (b) $\frac{2}{5}\sqrt{17}$ sq. unit

(c) $\frac{3}{5}\sqrt{17}$ sq. unit (d) $\frac{5}{3}\sqrt{17}$ sq. unit

15. A metal sphere is hung by a string fixed to a wall. The sphere is pushed away from the wall by a stick. The forces acting on the sphere are shown in the second diagram. Which of the following statements is wrong



(a) $P = W\tan\theta$ (b) $\vec{T} + \vec{P} + \vec{W} = 0$

(c) $T^2 = P^2 + W^2$ (d) $T = P + W$

ONE OR MORE THAN ONE CHOICE CORRECT

1. The angle between two vectors given by $6\bar{i} + 6\bar{j} - 3\bar{k}$ and $7\bar{i} + 4\bar{j} + 4\bar{k}$ is

(a) $\cos^{-1}\left(\frac{2}{3}\right)$ (b) $\cos^{-1}\left(\frac{5}{\sqrt{3}}\right)$ (c) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (d) $\sin^{-1}\left(\frac{\sqrt{5}}{3}\right)$

2. The quadratic equation $px^2 + 4x + 1 = 0, p > 0$ has real roots if $p =$

(a) 4 (b) 2 (c) 5 (d) $\frac{1}{2}$

3. If $x = k\sqrt{2}$ be a solution of the quadratic $x^2 + \sqrt{2}x - 4 = 0$, then $k =$

(a) – 1 (b) 1 (c) 2 (d) – 2

4. The quadratic equation $x^2 - 2x - 5 = 0$ have

(a) real roots (b) complex roots (c) repeated roots (d) distinct roots

5. Choose the trigonometric relations where values are equal

(a) cos 210° (b) $\sin(-60°)$

(c) $\cos 120° \cos 90° - \sin 120° \sin 90°$ (d) $\sin 200° \cos 100° + \cos 200° \sin 100°$

6. A situation may be described by using different sets of co-ordinate axes having different orientations. Which of the following do not depend on the orientation of the axes?
(a) the value of a scalar (b) component of a vector
(c) a vector (d) the magnitude of a vector

7. The x-component of the resultant of several vectors
(a) is equal to the sum of the x-components of the vectors
(b) may be smaller than the sum of the magnitudes of the vectors
(c) may be greater than the sum of the magnitude of the vectors
(d) may be equal to the sum of the magnitudes of the vectors.

8. The magnitude of the vector product of two vectors $\vec{A}$ and $\vec{B}$ may be
(a) greater than AB (b) equal to AB
(c) less than AB (d) equal to zero

9. Choose the correct statement about exponential number
(a) It is not a real number (b) It is an irrational number
(c) Its value is greater than 3 (d) Its value is between 2 and 3

10. Resultant of three non-zero vectors may be zero when
(a) they are coplanar only.
(b) they are non-coplanar.
(c) they represent three sides of a triangle both in magnitude and direction taken in a order.
(d) they represent three sides of a triangle both in magnitude and direction in any order.

EXERCISE – III

MATCH THE FOLLOWING

1. For two non-zero vectors $\vec{A}$ and $\vec{B}$, statements in column – I matches with only one value in column – II.

Column – I	**Column - II**
I. Angle between vectors $\vec{A}$ and $\vec{B}$	**A.** $\dfrac{\vec{A}\times\vec{B}}{\lvert\vec{A}\times\vec{B}\rvert}$
II. Unit vector perpendicular to $\vec{A}$ and $\vec{B}$	**B.** $\cos^{-1}\left(\dfrac{\vec{A}.\vec{B}}{\lvert\vec{A}\rvert\lvert\vec{B}\rvert}\right)$
III. Area of triangle whose two sides are given by vector $\vec{A}$ and $\vec{B}$	**C.** $\dfrac{1}{2}\lvert\vec{A}\times\vec{B}\rvert$
IV. If $\vec{A}$ and $\vec{B}$ are collinear then the value of $\lvert\vec{A}\times\vec{B}\rvert$	**D.** zero **E.** may be non zero

REASONING TYPE

Directions: Read the following questions and choose

(A) If both the statements are true and statement-2 is the correct explanation of **statement-1**.
(B) If both the statements are true but statement-2 is not the correct explanation of **statement-1**.
(C) If statement-1 is True and **statement-2** is False.
(D) If statement-1 is False **and statement-2** is True.

1. **Statement-1**: $\vec{A}\times\vec{B}$ is perpendicular to both $\vec{A}+\vec{B}$ as well as $\vec{A}-\vec{B}$.

Statement-2: $\vec{A}+\vec{B}$ as well as $\vec{A}-\vec{B}$ lie in the plane containing $\vec{A}$ and $\vec{B}$, but $\vec{A}\times\vec{B}$ lies perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

(a) (A) (b) (B) (c) (C) (d) (D)

2. **Statement-1**: Angle between $\hat{i}+\hat{j}$ and $\hat{i}$ is 45°

Statement-2: $\hat{i}+\hat{j}$ is equally inclined to both $\hat{i}$ and $\hat{j}$ and the angle between $\hat{i}$ and $\hat{j}$ is 90°

(a) (A) (b) (B) (c) (C) (d) (D)

3. **Statement-1**: Minimum number of non-equal vectors in a plane required to give zero resultant is three.

Statement-2: If $\vec{A}+\vec{B}+\vec{C}=\vec{O}$, then they must lie in one plane

(a) (A) (b) (B) (c) (C) (d) (D)

4. **Statement-1**: The cross product of a vector with itself is a null vector.

Statement-2: The cross-product of two vectors results in a vector quantity.

(a) (A) (b) (B) (c) (C) (d) (D)

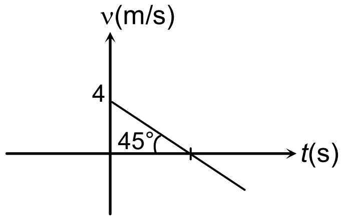
5. **Statement-1**: If $\vec{A}.\vec{B}=\vec{B}.\vec{C},$ then $\vec{A}$ may not always be equal to $\vec{C}$

Statement-2: The dot product of two vectors involves cosine of the angle between the two vectors.

(a) (A) (b) (B) (c) (C) (d) (D)

LINKED COMPREHENSION TYPE

The velocity time $(v-t)$ curve of a particle moving along a straight line is given



1. Initial velocity of particle is

(a) zero (b) 2 m/s (c) 4 m/s (d) 6 m/s

2. Velocity of particle at t = 6 s is

(a) 2 m/s (b) – 2 m/s (c) – 6 m/s (d) 6 m/s

3. Time when velocity of particle is zero

(a) 5 s (b) 2 s (c) 3 s (d) 4 s

EXERCISE – IV

SUBJECTIVE PROBLEMS

1. Find $\frac{dy}{dx}$ for the following:

(i) $y = 6x^5 + 4x^3 - 3x^2 + 2x - 7$

(ii) $y = \frac{(1+x)\sqrt{x}}{\sqrt[3]{x}}$

(iii) $y = 5 \cot x - 3 \ln x$

(iv) $y = 4^x - 3\operatorname{cosec} x$

(v) $y = e^x \cos x$

(vi) $y = \sqrt{x \sin x}$

2. Find the slope of the tangent to the curve $y = \sin x + \tan x$ at $x = \frac{\pi}{3}$.

3. The radius of a circular soap bubble is increasing at the rate of 0.2 cm/s. Find the rate of increase of its surface area, when the radius is 7 cm.

4. Find maximum and minimum values of $(2x^3 - 24x + 107)$ in the interval $[-3, 3]$.

5. Evaluate the following integrals:

(i) $\int (8 - x + 2x^3 - \frac{6}{x^3} + 2x^{-5} + 5x^{-1})dx$

(ii) $\int \sec x(\sec x + \tan x)dx$

6. Evaluate:

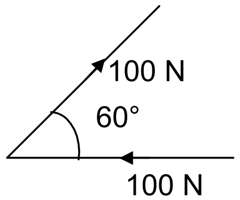
(i) $\int_1^4 \frac{dx}{\sqrt{x}}$

(ii) $\int_0^{\pi/6} \cos x \cos 2x \, dx$

7. Find the area bounded by the curve $y = \sec x \tan x$, x-axis and the lines $x = \frac{\pi}{3}$ and $x = \frac{\pi}{6}$.

8. A force of 10.5 N acts on a particle along a direction making an angle 37° with the vertical. Find the component of force in vertical direction.

9. In the given figure two numerically equal forces are acting as shown. What is the magnitude of their resultant?



10. Vector $\vec{A}$ has a magnitude of 5 units, $\vec{B}$ has a magnitude of 6 units and the cross product of $\vec{A}$ and $\vec{B}$ has a magnitude of 15 units. Find the angle between $\vec{A}$ and $\vec{B}$.

11. Force $F = 4\hat{i} + 5\hat{j}$ newton displaces a particle through $\vec{S} = 3\hat{i} + 6\hat{k}$ metre, then find the value of work done.

12. Vector A of magnitude 4 units is directed along the positive x-axis. Another vector B of magnitude 3 units lies in x-y plane and is directed along 30^0 with the positive x-axis in anticlock-wise direction. Find the value of $\vec{A} \times \vec{B}$.

13. If $\vec{a} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{b} = 7\hat{i} + 3\hat{j} + 6\hat{k}$, find a unit vector along $(\vec{a} \times \vec{b})$

14. Calculate the resultant of the following forces acting at a point, making use of resolution process.

(i) 100 $\sqrt{2}$ dynes along north-east

(ii) 980 $\sqrt{2}$ dynes along north-west

(iii) 1960 dyne along south

15. The greatest and the least resultant of two forces acting at a point is 11 N and 7 N respectively. If each force is increased by 3 N, find the resultant of new forces when acting at a point at an angle of 90° with each other.

ANSWERS

EXERCISE – I

1. (d)	2. (a)	3. (d)	4. (b)	5. (a)
6. (b)	7. (c)	8. (d)	9. (c)	10. (b)
11. (a)	12. (a)	13. (c)	14. (c)	15. (b)

EXERCISE – II

1. (b)	2. (b)	3. (b)	4. (d)	5. (b)
6. (b)	7. (d)	8. (c)	9. (c)	10. (c)
11. (b)	12. (b)	13. (c)	14. (a)	15. (d)

ONE OR MORE THAN ONE CHOICE CORRECT

1. (a,d)	2. (a,b,d)	3. (b,d)	4. (a,d)	5. (a,b,c,d)
6. (a,c,d)	7. (a,b,d)	8. (b,c,d)	9. (a,d)	10. (a,c)

EXERCISE – III

MATCH THE FOLLOWING

1. I – (B, E), II – (A, E), III – (C, E), (IV) – (D).

REASONING TYPE

1. (a)	2. (a)	3. (b)	4. (b)	5. (a)

LINKED COMPREHENSION TYPE

1. (c)	2. (b)	3. (d)

EXERCISE – IV

SUBJECTIVE PROBLEMS

1. (i) $30x^4 + 12x^2 - 6x + 2$ (ii) $\frac{1}{6}x^{-\frac{5}{6}} + \frac{7}{6}x^{\frac{1}{6}}$

(iii) $-5\text{cosec}^2 x - \frac{3}{x}$ (iv) $4^x \ln 4 + 3 \text{ cosec } x \cot x$

(v) $-\sin x\, e^x + \cos x\, e^x$ (vi) $\frac{x\cos x + \sin x}{2\sqrt{x \sin x}}$

2. $\frac{9}{2}$

3. 35.2 cm^2/s

4. 139, 75 respectively

5. (i) $8x - \frac{x^2}{2} + \frac{x^4}{2} + \frac{3}{x^2} - \frac{x^{-4}}{2} + 5\ln x + c$

(ii) $\tan x + \sec x + c$

6. (i) 2 (ii) $\frac{5}{12}$

7. $\left(2 - \frac{2}{\sqrt{3}}\right)$ sq. units.

8. 8.4 N

9. 100 N

10. $\theta = 30°$

11. 12 joule

12. $6\hat{k}$

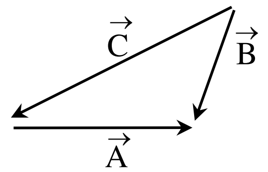
13. $\frac{9}{\sqrt{731}}\hat{i} + \frac{17}{\sqrt{731}}\hat{j} - \frac{19}{\sqrt{731}}\hat{k}$

14. 1.244×10^3 dyne, south-west direction.

15. 13 N

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 1

Q.1 For the figure –



(1) $\vec{A}+\vec{B}=\vec{C}$ (2) $\vec{B}+\vec{C}=\vec{A}$

(3) $\vec{C}+\vec{A}=\vec{B}$ (4) $\vec{A}+\vec{B}+\vec{C}=0$

Q.2 Two forces of 4 dyne and 3 dyne act upon a body. The resultant force on the body can only be –

(1) more than 3 dynes

(2) more than 4 dynes

(3) between 3 and 4 dynes

(4) between 1 and 7 dynes

Q.3 A force of 6 kg and another of 8 kg can be applied together to produce the effect of a single force of-

(1) 1kg (2) 11kg (3) 15 kg (4) 20 kg

Q.4 Which of the sets given below may represent the magnitudes of three vectors adding to zero ?

(1) 2, 4, 8 (2) 4, 8, 16

(3) 1, 2, 1 (4) 0.5, 1, 2

Q.5 Two vectors have magnitudes 3 unit and 4 unit respectively. What should be the angle between them if the magnitude of the resultant is -

(i) 1 unit (ii) 5 unit (iii) 7 unit

(1) 180º, 90º, 0º (2) 80º, 70º, 0º

(3) 90º, 170º, 50º (4) None of these

Q.6 A blind person after walking 10 steps in one direction, each of length 80 cm, turns randomly to the left or to the right by 90º. After walking a total of 40 steps the maximum possible displacement of the person from his starting position could be -

(1) 320 m (2) 32 m

(3) 16/$\sqrt{2}$ m (4) 16$\sqrt{2}$ m

Q.7 If the angle between vector $\vec{a}$ and $\vec{b}$ is an acute angle, then the difference $\vec{a}-\vec{b}$ is -

(1) the main diagonal of the parallelogram

(2) the minor diagonal of the parallelogram

(3) any of the above

(4) none of the above

Q.8 What is the resultant of three coplanar forces: 300 N at 0°, 400 N at 30º and 400 N at 150º ?

(1) 500 N (2) 700 N (3) 1100N (4) 300 N

Q.9 Two forces, F_1 and F_2 are acting on a body. One force is double that of the other force and the resultant is equal to the greater force. Then the angle between the two forces is -

(1) $\cos^{-1}(1/2)$ (2) $\cos^{-1}(-1/2)$

(3) $\cos^{-1}(-1/4)$ (4) $\cos^{-1}(1/4)$

Q.10 If the magnitudes of the vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 6, 8, 10 units respectively and if $\vec{A} + \vec{B} = \vec{C}$, then the angle between $\vec{A}$ and $\vec{C}$ is -

(1) $\pi/2$

(2) arc cos (0. 6)

(3) arc tan (0.75)

(4) $\pi/4$

Q.11 Angle between $(\vec{P} + \vec{Q})$ and $(\vec{P} - \vec{Q})$ will be-

(1) 0º only

(2) 90º only

(3) 180º only

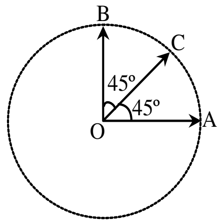
(4) between 0º and 180º (both the values inclusive)

Q.12 A particle is moving in a circle of radius r centre at O with constant speed v the change in velocity moving from A to B ($\angle AOB = 40º$) is -

(1) 2v cos 40º (2) 2v sin 40º

(3) 2v cos 20º (4) 2v sin 20º

Q.13 The three vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ have the same magnitude R. Then the sum of these vectors have magnitude –



(1) R (2) $\sqrt{2}$ R (3) 3R (4) (1+$\sqrt{2}$)R

Q.14 What displacement must be added to the displacement $25\hat{i} - 6\hat{j}$ m to give a displacement of 7.0 m pointing in the x-direction ?

(1) $18\hat{i} - 6\hat{j}$ (2) $32\hat{i} - 13\hat{j}$

(3) $-18\hat{i} + 6\hat{j}$ (4) $-25\hat{i} + 13\hat{j}$

Q.15 Two constant forces $\vec{F}_1 = 2\hat{i} - 3\hat{j} + 3\hat{k}$ (N) and $\vec{F}_2 = \hat{i} + \hat{j} - 2\hat{k}$ (N) act on a body and displace it from the position $\vec{r}_1 = \hat{i} + 2\hat{j} - 2\hat{k}$ (m) to the position $\vec{r}_2 = 7\hat{i} + 10\hat{j} + 5\hat{k}$ (m). What is the work done ?

(1) 9 Joule (2) 41 Joule

(3) –3 Joule (4) None of these

Q.16 Two vectors $\vec{A}$ and $\vec{B}$ lie in X-Y plane. The vector B is perpendicular to vector $\vec{A}$. If $\vec{A} = \hat{i} + \hat{j}$, then $\vec{B}$ may be -

(1) $\hat{i} - \hat{j}$ (2) $-\hat{i} + \hat{j}$

(3) $-2\hat{i} + 2\hat{j}$ (4) Any of the above

Q.17 The two vectors $\vec{A} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{B} = 7\hat{i} - 5\hat{j} - 3\hat{k}$ are -

(1) parallel (2) perpendicular

(3) anti-parallel (4) none of these

Q.18 Two vectors $\vec{P} = 2\hat{i} + b\hat{j} + 2\hat{k}$ and $\vec{Q} = \hat{i} + \hat{j} + \hat{k}$ will be perpendicular if -

(1) b = 0 (2) b = 1

(3) b = 2 (4) b = – 4

Q.19 A vector perpendicular to $(4\hat{i} - 3\hat{j})$ is –

(1) $4\hat{i}+3\hat{j}$ (2) $7\hat{k}$

(3) $6\hat{i}$ (4) $3\hat{i}-4\hat{j}$

Q.20 Angle that the vector $\vec{A} = 2\hat{i} + 3\hat{j}$ makes with y-axis is –

(1) $\tan^{-1} 3/2$ (2) $\tan^{-1} 2/3$

(3) $\sin^{-1} 2/3$ (4) $\cos^{-1} 3/2$

Q.21 A vector $\vec{A}$ points. vertically upward and, $\vec{B}$ points towards north. The vector product $\vec{A} \times \vec{B}$ is-

(1) along west

(2) along east

(3) zero

(4) vertically downward

Q.22 The linear velocity of a rotating body is given by $\vec{v} = \vec{\omega} \times \vec{r}$, where $\vec{\omega}$ is the angular velocity and $\vec{r}$ is the radius vector. The angular velocity of a body $\vec{\omega} = \hat{i}-2\hat{j}+2\hat{k}$ and their radius vector $\vec{r} = 4\hat{j} - 3\hat{k}$, |**v**| is -

(1) $\sqrt{29}$ units (2) 31 units

(3) $\sqrt{37}$ units (4) $\sqrt{41}$ units

Q.23 $0.4\hat{i} + 0.8\hat{j} + c\hat{k}$ represents a unit vector, when c is -

(1) 0.2 (2) $\sqrt{0.2}$

(3) $\sqrt{0.8}$ (4) 0

Q.24 A vector is not changed if -

(1) It is rotated through an arbitrary angle

(2) It is multiplied by an arbitrary scale

(3) It is cross multiplied by a unit vector

(4) It is a slide parallel to itself

Q.25 The component of a vector is -

(1) always less than its magnitude

(2) always greater than its magnitude

(3) always equal to its magnitude

(4) none of these

Q.26 If $\vec{A} = \vec{B} + \vec{C}$ and the magnitudes $\vec{A}, \vec{B}$ and $\vec{C}$ are 5, 4 and 3 units, the angle between $\vec{A}$ and $\vec{C}$ is-

(1) $\cos^{-1}\left(\frac{3}{5}\right)$ (2) $\cos^{-1}\left(\frac{4}{5}\right)$

(3) $\frac{\pi}{2}$ (4) $\sin^{-1}\left(\frac{3}{4}\right)$

Q.27 The resultant of $\vec{A}$ and $\vec{B}$ makes an angle α with $\vec{A}$ and β with $\vec{B}$, then -

(1) $\alpha < \beta$

(2) $\alpha < \beta$ if A < B

(3) $\alpha < \beta$ if A > B

(4) $\alpha < \beta$ if A = B

Q.28 I started walking down a road to day-break facing the sun. After walking for some-time, I turned to my left, then I turned to the right once again. In which direction was I going then ?

(1) East (2) North-west

(3) North-east (4) South

Q.29 Minimum number of unequal forces whose vector sum can equal to zero is -

(1) two (2) three

(3) four (4) any

Q.30 How many minimum number of vectors in different planes can be added to give zero resultant ?

(1) 2 (2) 3

(3) 4 (4) 5

Q.31 Following sets of three forces act on a body. Whose resultant cannot be zero ?

(1) 10, 10, 10 (2) 10, 10, 20

(3) 10, 20, 20 (4) 10, 20, 40

Q.32 Following forces start acting on a particle at rest at the origin of the co-ordinate system simultaneously

$\vec{F}_1 = -4\hat{i} - 5\hat{j} + 5\hat{k}$ $\quad\quad$ $\vec{F}_2 = 5\hat{i} + 8\hat{j} + 6\hat{k}$

$\vec{F}_3 = -3\hat{i} + 4\hat{j} - 7\hat{k}$ $\quad\quad$ $\vec{F}_4 = 2\hat{i} - 3\hat{j} - 2\hat{k}$

then the particle will move -

(1) In x - y plane

(2) In y - z plane

(3) In x - z plane

(4) Along x-axis

Q.33 If $|\vec{A}+\vec{B}| = |\vec{A}| = |\vec{B}|$, the angle between $\vec{A}$ and $\vec{B}$ is -

(1) 60º (2) 0º (3) 120º (4) 90º

Q.34 The vector $\vec{A}$ and $\vec{B}$ are such that-

$\vec{A}+\vec{B} = \vec{A}-\vec{B}$

(1) $\vec{A}+\vec{B} = 0$ (2) $\vec{A}-\vec{B} = 0$

(3) $\vec{A} = 0$ (4) $\vec{B} = 0$

Q.35 In an equilateral ΔABC, AL, BM and CN are medians. Forces along BC and BA represented by them will have a resultant represented by -

(1) 2AL (2) 2BM

(3) 2CN (4) AC

Q.36 Two forces each of magnitude F have a resultant of the same magnitude F. The angle between the two forces is -

(1) 45º (2) 120º

(3) 150º (4) 60º

Q.37 A particle is moving on a circular path with constant speed v. What is the change in its velocity after it has described an angle of 60º ?

(1) v $\sqrt{2}$ (2) v $\sqrt{3}$

(3) v (4) 2 v

Q.38 Which of the following sets of displacements might be capable of returning a car to its starting point ?

(1) 4, 6, 8 and 15 km

(2) 10, 30, 50 and 120 km

(3) 5,10, 30 and 50 km

(4) 50, 50, 75 and 200 km

Q.39 The magnitude of the vector product of two vectors $\vec{A}$ and $\vec{B}$ may be -

(a) Greater than AB (b) Equal to AB

(c) Less than AB (d) Equal to Zero

(1) a, b, c (2) b, c, d

(3) a, c, d (4) a, b, d

Q.40 Three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ satisfy the relation $\vec{A}.\vec{B}$ = 0 and $\vec{A}.\vec{C}$ = 0. The vector $\vec{A}$ is parallel to -

(1) $\vec{B}$ (2) $\vec{C}$

(3) $\vec{B}.\vec{C}$ (4) $\vec{B}\times\vec{C}$

Q.41 The angle between the two vectors $-2\hat{i}+3\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}-4\hat{k}$ is -

(1) 0º (2) 90º

(3) 180º (4) None

Q.42 What is the angle between $(\vec{P}+\vec{Q})$ and $(\vec{P}\times\vec{Q})$?

(1) 0 (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{4}$ (4) π

Q.43 A vector $\vec{A}$ points vertically upward and $\vec{B}$ points towards north. The vector product $\vec{A}\times\vec{B}$ is-

(1) along west

(2) along east

(3) zero

(4) vertically downward

Q.44 A vector is along the positive x-axis. If its vector product with another vector $\vec{F}_2$ is zero, then $\vec{F}_2$ could be -

(1) $4\hat{j}$

(2) $-(\hat{i}+\hat{j})$

(3) $(\hat{j}+\hat{k})$

(4) $(-4\hat{i})$

Q.45 For any two vectors $\vec{A}$ and $\vec{B}$, if $\vec{A}.\vec{B} = |\vec{A}\times\vec{B}|$, the magnitude of $\vec{C} = \vec{A}+\vec{B}$ is equal to -

(1) $\sqrt{A^2+B^2}$

(2) A + B

(3) $\sqrt{A^2+B^2+\frac{AB}{\sqrt{2}}}$

(4) $\sqrt{A^2+B^2+\sqrt{2}AB}$

Q.46 Which of the following is not true ? If $\vec{A} = 3\hat{i}+4\hat{j}$ and $\vec{B} = 6\hat{i}+8\hat{j}$ where A and B are the magnitudes of $\vec{A}$ and $\vec{B}$?

(1) $\vec{A}\times\vec{B} = 0$ (2) $\frac{A}{B} = \frac{1}{2}$

(3) $\vec{A}.\vec{B} = 48$ (4) A = 5

Q.47 A unit vector along the direction $\hat{i}+\hat{j}+\hat{k}$ has a magnitude -

(1) $\sqrt{3}$ (2) $\sqrt{2}$

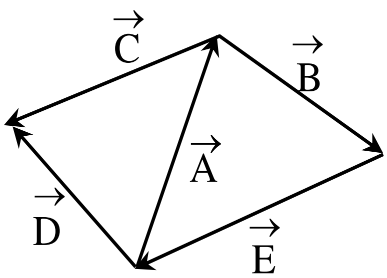
(3) 1 (4) 0

Q.48 If vectors $\vec{A} = \hat{i} + 2\hat{j} + 4\hat{k}$ and $\vec{B} = 5\hat{i}$ represent the two sides of a triangle, then the third side of the triangle has length equal to -

(1) $\sqrt{56}$ (2) $\sqrt{21}$

(3) 5 (4) 6

Q.49 For figure the correct relation is-



(1) $\vec{A}+\vec{B}+\vec{E} = 0$ (2) $\vec{C}-\vec{D} = -\vec{A}$

(3) $\vec{B}+\vec{E}-\vec{C} = -\vec{D}$ (4) all of the above

Q.50 The position vectors of points $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$ are

$\vec{A} = 3\hat{i}+4\hat{j}+5\hat{k}$, $\vec{B} = 4\hat{i}+5\hat{j}+6\hat{k}$,

$\vec{C} = 7\hat{i}+9\hat{j}+3\hat{k}$ and $\vec{D} = 4\hat{i}+6\hat{j}$

Then the displacement vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are -

(1) perpendicular

(2) parallel

(3) anti-parallel

(4) inclined at an angle of 60º

Q.51 Let the angle between two non zero vectors $\vec{A}$ and $\vec{B}$ be 120º and its resultant be $\vec{C}$ -

(a) C must be equal to $|\vec{A} - \vec{B}|$

(b) C must be less than $|\vec{A} - \vec{B}|$

(c) C must be greater than $|\vec{A} - \vec{B}|$

(d) C may be equal to $|\vec{A} - \vec{B}|$

then the correct statement is -

(1) a (2) b (3) c (4) d

Q.52 At what angle must the two forces (x + y) and (x – y) act so that the resultant may be $\sqrt{x^2+y^2}$?

(1) $\cos^{-1}\left[-\frac{x^2+y^2}{2(x^2-y^2)}\right]$

(2) $\cos^{-1}\left[-2\frac{(x^2-y^2)}{x^2+y^2}\right]$

(3) $\cos^{-1}\left[-\frac{x^2+y^2}{x^2-y^2}\right]$

(4) $\cos^{-1}\left[-\frac{x^2-y^2}{x^2+y^2}\right]$

Q.53 Three vectors $\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{B} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{C} = 3\hat{i} - 4\hat{j} - 4\hat{k}$ are sides of an-

(1) equilateral triangle

(2) right angled triangle

(3) isosceles triangle

(4) none of the above

Q.54 The area of a parallelogram formed from the vectors $\vec{A} = \hat{i}-2\hat{j}+3\hat{k}$ and $\vec{B} = 3\hat{i}-2\hat{j}+\hat{k}$ as adjacent sides is -

(1) $8\sqrt{3}$ units (2) 64 units

(3) 32 units (4) $4\sqrt{6}$ units

Q.55 Out of addition, subtraction, dot product and cross product, the following operations are commutative -

(1) dot and cross products

(2) addition and subtraction

(3) subtraction and cross product

(4) addition and dot product

Q.56 The angle between two vector $\vec{A}$ and $\vec{B}$ is θ. Then the magnitude of the product $\vec{A}.(\vec{B}\times\vec{A})$ is -

(1) $A^2 B$ (2) $A^2B \sin\theta$

(3) $A^2B \sin\theta \cos\theta$ (4) Zero

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM - 2

Q.1 Angular momentum is-

(1) Axial vector (2) Polar vector

(3) Scalar (4) None of these

Q.2 A force vector applied on a mass is represented as $\vec{F}=6\hat{i}-8\hat{j}+10\hat{k}$ and the mass accelerates with 1 m/s^2. What will be the mass of the body ?

(1) $10\sqrt{2}$ kg (2) $2\sqrt{10}$ kg

(3) 10 kg (4) 20 kg

Q.3 Find the torque of a force $\vec{F}=2\hat{i}+\hat{j}+4\hat{k}$ acting at the point $\vec{r}=7\hat{i}+3\hat{j}+\hat{k}$.

(1) $14\hat{i}-38\hat{j}+16\hat{k}$ (2) $4\hat{i}+4\hat{j}+6\hat{k}$

(3) $-14\hat{i}+38\hat{j}-16\hat{k}$ (4) $11\hat{i}-26\hat{j}+\hat{k}$

Q.4 If a unit vector is represented by $0.5\hat{i}-0.8\hat{j}+c\hat{k}$, then the value of 'c' is-

(1)1 (2) $\sqrt{0.11}$

(3) $\sqrt{0.01}$ (4) $\sqrt{0.39}$

Q.5 For a body, angular velocity $(\vec{\omega})$ = $\hat{i}-2\hat{j}+3\hat{k}$ and radius vector $(\vec{r})$ = $\hat{i}+\hat{j}+\hat{k}$?

(1) $-5\hat{i}+2\hat{j}+3\hat{k}$ (2) $-5\hat{i}+2\hat{j}-3\hat{k}$

(3) $-5\hat{i}-2\hat{j}+8\hat{k}$ (4) $-5\hat{i}-2\hat{j}-3\hat{k}$

Q.6 What is the value of linear velocity, if $\vec{\omega}=3\hat{i}-4\hat{j}+\hat{k}$ and $\vec{r}=5\hat{i}-6\hat{j}+6\hat{k}$?

(1) $4\hat{i}-13\hat{j}+6\hat{k}$ (2) $6\hat{i}-2\hat{j}+3\hat{k}$

(3) $6\hat{i}-2\hat{j}+8\hat{k}$ (4) $-18\hat{i}-13\hat{j}+2\hat{k}$

Q.7 If $\vec{F}=(60\hat{i}+15\hat{j}-3\hat{k})\,\text{N}$ & $\vec{v}=(2\hat{i}-4\hat{j}+5\hat{k})$ m/s then instantaneous power is-

(1) 195 watt (2) 45 watt

(3) 75 watt (4) 100 watt

Q.8 If $|\vec{A}+\vec{B}|$ = $|\vec{A}|$ = $|\vec{B}|$ then angle between $\vec{A}$ and $\vec{B}$ will be-

(1) 90º (2) 120º

(3) 0º (4) 60º

Q.9 The vector sum of two forces is perpendicular to their vector difference. In that case, the force-

(1) Are equal to each other

(2) Are equal to each other in magnitude

(3) Are not equal to each other in magnitude

(4) Cannot be predicted

Q.10 If $|\vec{A}\times\vec{B}| = \sqrt{3}\,\vec{A}.\vec{B}$, then the value of $|\vec{A}+\vec{B}|$ is-

(1) $\left(A^2+B^2+\dfrac{AB}{\sqrt{3}}\right)^{1/2}$

(2) A + B

(3) $(A^2 + B^2 + \sqrt{3}\,AB)^{1/2}$

(4) $(A^2 + B^2 + AB)^{1/2}$

Q.11 If a vector $(2\hat{i}+3\hat{j}+8\hat{k})$ is perpendicular to the vector $(4\hat{j}-4\hat{i}+\alpha\hat{k})$, then the value of α is-

(1) –1 (2) 1/2

(3) –1/2 (4) 1

Q.12 If the angle between the vector $\vec{A}$ and $\vec{B}$ is θ, the value of the product $(\vec{B}\times\vec{A}).\vec{A}$ is equal to-

(1) $BA^2\cos\theta$ (2) $BA^2\sin\theta$

(3) $BA^2\sin\theta\cos\theta$ (4) zero

Q.13 The vectors $\vec{A}$ and $\vec{B}$ are such that $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$. The angle between vectors $\vec{A}$ and $\vec{B}$ is-

(1) 90º (2) 60º

(3) 75º (4) 45º

Q.14 The angle between the two vectors $\vec{A}=3\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{B}=3\hat{i}+4\hat{j}-5\hat{k}$ will be-

(1) zero (2) 180º

(3) 90º (4) 45º

Q.15 The forces, which meet at one point but their lines of action do not lie in one plane, are called-

(1) non-coplanar and non-concurrent forces

(2) coplanar and non-concurrent forces

(3) non-coplanar and concurrent forces

(4) coplanar and concurrent forces

Q.16 What happens, when we multiply a vector by (–2) ?

(1) direction reverses and unit changes

(2) direction reverses and magnitude is doubled

(3) direction remains unchanged and unit changes

(4) none of these

Q.17 If $\vec{P}.\vec{Q}$ = PQ, then angle between $\vec{P}$ and $\vec{Q}$ is-

(1) 0º (2) 30º

(3) 45º (4) 60º

Q.18 Two vectors of equal magnitude have a resultant equal to either of them in magnitude. The angle between them is-

(1) 60º (2) 90º

(3) 105º (4) 120º

Q.19 A force of $(3\hat{i}+4\hat{j})$ Newton acts on a body and displaces it by $(3\hat{i}+4\hat{j})$ metre. The work done by the force is-

(1) 10 J (2) 12 J

(3) 19 J (4) 25 J

Q.20 The vector $\vec{P}=a\hat{i}+a\hat{j}+3\hat{k}$ and $\vec{Q}=a\hat{i}-2\hat{j}-\hat{k}$ are perpendicular to each other. The positive value of 'a' is-

(1) 3 (2) 2

(3) 1 (4) zero

Q.21 The direction of the angular velocity vector is along-

(1) the tangent to the circular path

(2) the inward radius

(3) the outward radius

(4) the axis of rotation

Q.22 $\vec{A}$ and $\vec{B}$ are two vectors and θ is the angle between them, if $|\vec{A}\times\vec{B}|$ = $\sqrt{3}(\vec{A}.\vec{B})$ the value of θ is-

(1) 90º (2) 60º

(3) 45º (4) 30º

Q.23 A body is moving with velocity 30 m/s towards east. After 10 seconds its velocity becomes 40 m/s towards north. The average acceleration of the body is :

(1) 5 m/s^2 (2) 1 m/s^2

(3) 7 m/s^2 (4) $\sqrt{7}$ m/s^2

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 3

Q.1 When forces F_1, F_2, F_3 are acting on a particle of mass m such that F_2 and F_3 are mutually perpendicular, then the particle remains stationary. If the force F_1 is now removed then the acceleration of the particle is –

(1) $\frac{F_1}{m}$ (2) $\frac{F_2F_3}{mF_1}$

(3) $\frac{(F_2 - F_3)}{m}$ (4) $\frac{F_2}{m}$

Q.2 Two forces are such that the sum of their magnitudes is 18 N and their resultant 12 N is perpendicular to the smaller force. Then the magnitudes of the forces are –

(1) 12 N, 6 N (2) 13 N, 5 N

(3) 10 N, 8 N (4) 16 N, 2 N

Q.3 If $\vec{A}\times\vec{B}=\vec{B}\times\vec{A}$, then the angle between A and B is –

(1) π (2) $\pi/3$

(3) $\pi/2$ (4) $\pi/4$

Q.4 The angle made by the vector $\vec{A}=\hat{i}+\hat{j}$ with x-axis is-

(1) 90º (2) 45º

(3) 22.5º (4) 30º

Q.5 If the sum of two unit vectors is a unit vector, then the magnitude of their difference is-

(1) $\sqrt{2}$ (2) $\sqrt{3}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{5}$

Q.6 Which of the following is a vector quantity ?

(1) Temperature (2) Surface tension

(3) Calorie (4) Force

Q.7 The magnitudes of vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are respectively 12, 5 and 13 units and $\vec{A}+\vec{B}=\vec{C}$, then the angle between $\vec{A}$ and $\vec{B}$ is-

(1) 0 (2) 45º

(3) $\pi/2$ (4) $\pi/4$

Q.8 The angle between two vectors $(2\hat{i}+3\hat{j}+\hat{k})$ and $(-3\hat{i}+6\hat{k})$ is-

(1) 0º (2) 45º

(3) 60º (4) 90º

Q.9 Let $\vec{A}=\hat{i}A\cos\theta+\hat{j}A\sin\theta$, be any vector. Another vector $\vec{B}$ which is normal to $\vec{A}$ is-

(1) $\hat{i}B\cos\theta+\hat{j}B\sin\theta$ (2) $\hat{i}B\sin\theta+\hat{j}B\cos\theta$

(3) $\hat{i}B\sin\theta-\hat{j}B\cos\theta$ (4) $\hat{i}A\cos\theta-\hat{j}A\sin\theta$

Q.10 Which of the following is a scalar quantity ?

(1) Displacement (2) Electric field

(3) Acceleration (4) Work

Q.11 The sum of magnitudes of two forces acting at a point is 16N. If the resultant force is 8N and its direction is perpendicular to smaller force, then the forces are-

(1) 6N and 10N (2) 8N and 8N

(3) 4N and 12N (4) 2N and 14N

Q.12 If vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ have magnitude 5, 12 and 13 units and $\vec{P}+\vec{Q}=\vec{R}$, the angle between $\vec{Q}$ and $\vec{R}$ is-

(1) $\cos^{-1}\left(\frac{5}{12}\right)$ (2) $\cos^{-1}\left(\frac{5}{13}\right)$

(3) $\cos^{-1}\left(\frac{12}{13}\right)$ (4) $\cos^{-1}\left(\frac{2}{13}\right)$

Q.13 If two numerically equal forces P and P acting at a point produce a resultant force of magnitude P itself, then the angle between the two original forces is-

(1) 0º (2) 60º

(3) 90º (4) 120º

Q.14 If $\vec{A}+\vec{B}=\vec{C}$ and $|\vec{A}|=|\vec{B}|=|\vec{C}|$ then the angle between $\vec{A}$ and $\vec{B}$ is-

(1) 45º (2) 60º

(3) 90º (4) 120º

Q.15 The angle between two vectors given by $(6\hat{i}+6\hat{j}-3\hat{k})$ and $(7\hat{i}+4\hat{j}+4\hat{k})$ is-

(1) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (2) $\cos^{-1}\left(\frac{5}{\sqrt{3}}\right)$

(3) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (4) $\sin^{-1}\left(\frac{\sqrt{5}}{3}\right)$

Q.16 Which of the following vector identities is false ?

(1) $\vec{P}+\vec{Q}=\vec{Q}+\vec{P}$ (2) $\vec{P}+\vec{Q}=\vec{Q}\times\vec{P}$

(3) $\vec{P}.\vec{Q}=\vec{Q}.\vec{P}$ (4) $\vec{P}\times\vec{Q}\neq\vec{Q}\times\vec{P}$

Q.17 Which of the following is a scalar quantity ?

(1) current (2) velocity

(3) force (4) acceleration

Q.18 If $\hat{n}=a\hat{i}+b\hat{j}$ is perpendicular to the vector $(\hat{i}+\hat{j})$, then the value of 'a' and 'b' may be-

(1) 1, 0 (2) –2, 0

(3) 3, 0 (4) $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

Q.19 Which of the following pair of forces will never give resultant force of 2 N ?

(1) 2N and 2N (2) 1N and 1N

(3) 1N and 3N (4) 1N and 4N

Q.20 The vector $\vec{B}$ is directed vertically upwards and the vector $\vec{C}$ points towards south, then $\vec{B}\times\vec{C}$ will be-

(1) in west

(2) in east

(3) zero

(4) vertically downwards

Q.21 A vector of length ℓ is turned through the angle θ about its tail. What is the change in the position vector of its head ?

(1) ℓ cos(θ/2) (2) 2ℓ sin(θ/2)

(3) 2 ℓ cos(θ/2) (4) ℓ sin(θ/2)

Q.22 Force 3N, 4N and 12N act at a point in mutually perpendicular directions. The magnitude of the resultant force is-

(1) 19 N (2) 13 N (3) 11 N (4) 5 N

Q.23 The magnitude of a vector cannot be-

(1) positive (2) unity

(3) negative (4) zero

Q.24 The angle between vectors $(\vec{A}\times\vec{B})$ and $(\vec{B}\times\vec{A})$ is-

(1) π rad (2) $\frac{\pi}{2}$ rad

(3) $\frac{\pi}{4}$ rad (4) zero

Q.25 $\vec{A}$ and $\vec{B}$ are two vectors. Now indicate the wrong statement in the following-

(1) $\vec{A}.\vec{B}=\vec{B}.\vec{A}$ (2) $\vec{A}+\vec{B}=\vec{B}+\vec{A}$

(3) $\vec{A}\times\vec{B}=\vec{B}\times\vec{A}$ (4) $\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$

Q.26 The unit vector parallel to the resultant of the vectors $\vec{A}=4\hat{i}+3\hat{j}+6\hat{k}$ and $\vec{B}=-\hat{i}+3\hat{j}-8\hat{k}$ is-

(1) $\frac{1}{7}[3\hat{i}+6\hat{j}-2\hat{k}]$ (2) $\frac{1}{7}[3\hat{i}+6\hat{j}+2\hat{k}]$

(3) $\frac{1}{49}[3\hat{i}+6\hat{j}+2\hat{k}]$ (4) $\frac{1}{49}[3\hat{i}+6\hat{j}-2\hat{k}]$

Q.27 A vector $\vec{A}$ points vertically upwards and $\vec{B}$ points towards north. The vector product $\vec{A}\times\vec{B}$ is-

(1) zero (2) along west

(3) along east (4) vertically downward

Q.28 Which of the following sets of concurrent forces may be in equilibrium ?

(1) F_1 = 3N, F_2 = 5N, F_3 = 1N

(2) F_1 = 3N, F_2 = 5N, F_3 = 9N

(3) F_1 = 3N, F_2 = 5N, F_3 = 6N

(4) F_1 = 3N, F_2 = 5N, F_3 = 15N

Q.29 If three vectors satisfy the relation $\vec{A}.\vec{B}$ = 0 and $\vec{A}.\vec{C}$ = 0, then $\vec{A}$ can be parallel to-

(1) $\vec{C}$ (2) $\vec{B}$

(3) $\vec{B}\times\vec{C}$ (4) $\vec{B}.\vec{C}$

Q.30 What is the projection of $3\hat{i}+4\hat{k}$ on the y-axis ?

(1) 3 (2) 4

(3) 5 (4) zero

Q.31 Square of the resultant of two forces of equal magnitude is equal to three times the product of their magnitude. The angle between them is-

(1) 0º (2) 45º

(3) 60º (4) 90º

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 4

These questions of two statements each, printed as Assertion and Reason. While answering these Questions you are required to choose any one of the following four responses.
(1) If both Assertion & Reason are true & the Reason is a correct explanation of the Assertion.
(2) If both Assertion and Reason are true but Reason is not a correct explanation of the Assertion.
(3) If Assertion is true but the Reason is false.
(4) If Assertion & Reason both are false.

Q.1 **Assertion :** If the initial and final positions coincide, the displacement is a null vector.

Reason : A physical quantity can not be called a vector, if its magnitude is zero.

Q.2 **Assertion :** A vector quantity is a quantity that has both magnitude and a direction and obeys the triangle law of addition or equivalently the parallelogram law of addition.

Reason : The magnitude of the resultant vector of two given vectors can never be less than the magnitude of any of the given vector.

Q.3 **Assertion :** The direction of a zero (null) vector is indeterminate.

Reason : We can have $\vec{A}\times\vec{B}=\vec{A}.\vec{B}$ with $A \neq 0$ and $B \neq 0$.

Q.4 **Assertion :** If the rectangular components of a force are 24 N and 7 N, then the magnitude of the force is 25 N.

Reason : If $|\vec{A}|=|\vec{B}| = 1$ then $|\vec{A}\times\vec{B}|^2 + |\vec{A}.\vec{B}|^2 = 1$.

Q.5 **Assertion :** If three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ satisfy the relation $\vec{A}.\vec{B}$ **= 0** and $\vec{A}.\vec{C}$ **= 0** then the vector $\vec{A}$ may be parallel to $\vec{B}\times\vec{C}$.

Reason : If $\vec{A}+\vec{B}=\vec{R}$ and **A + B = R**, then angle between $\vec{A}$ and $\vec{B}$ is zero.

Q.6 **Assertion :** The angle between vectors $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ is π radian.

Reason : $\vec{B}\times\vec{A} = -\vec{A}\times\vec{B}$

Q.7 **Assertion :** The minimum number of vectors of unequal magnitude required to produce zero resultant is three.

Reason : Three vectors of unequal magnitude which can be represented by the three sides of a triangle taken in order, produce zero resultant.

Q.8 **Assertion :** A vector can have zero magnitude if one of its components is not zero.

Reason : Scalar product of two vectors cannot be a negative quantity.

Q.9 **Assertion :** The angle between the two vectors $(\hat{i}+\hat{j})$ and $(\hat{j}+\hat{k})$ is $\frac{\pi}{3}$ radian.

Reason : Angle between two vectors $\vec{A}$ and $\vec{B}$ is given by $\theta = \cos^{-1}\left(\frac{\vec{A}.\vec{B}}{AB}\right)$.

Q.10 **Assertion :** Distance is a scalar quantity.

Reason : Distance is the length of path traversed.

Q.11 **Assertion :** If position vector is given by $\vec{r} = \sin t\,\hat{i} + \cos t\,\hat{j} - 7t\,\hat{k}$, then magnitude of acceleration $|\vec{a}|$ = 1.

Reason : The angles which the vector $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ makes with the coordinate axes are given by $\cos\alpha = \frac{A_1}{A}$, $\cos\beta = \frac{A_2}{A}$ and $\cos\gamma = \frac{A_3}{A}$.

Q.12 **Assertion :** Adding a scalar to a vector of the same dimensions is a meaningful algebraic operation.

Reason : The displacement can be added with distance.

Q.13 **Assertion :** Vector $(\hat{i}+\hat{j}+\hat{k})$ is perpendicular to $(\hat{i}-2\hat{j}+\hat{k})$.

Reason : Two non-zero vectors are perpendicular if their dot product is equal to zero.

Q.14 **Assertion :** The dot product of one vector with another vector may be a scalar or a vector.

Reason : If the product of two vectors is a vector quantity, then product is called a dot product.

Q.15 **Assertion :** A physical quantity can be regarded as a vector, if magnitude as well as direction is associated with it.

Reason : A physical quantity can be regarded as a scalar quantity, if it is associated with magnitude only.

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 1 (ANSWERS)

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	3	4	2	3	1	4	2	1	3	2	4	4	4	3	1	4	2	4	3	2
Ques.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	1	1	2	4	1	1	3	1	2	3	4	2	3	4	2	2	3	1	2	4
Ques.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56				
Ans.	2	2	1	4	4	3	3	1	4	3	2	1	2	4	4	4				

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 2 (ANSWERS)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	1	1	4	2	1	4	2	2	2	4	3	4	1	3	3
Q.No.	16	17	18	19	20	21	22	23							
Ans.	2	1	4	4	1	4	2	1							

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 3 (ANSWERS)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	1	2	1	2	2	4	3	4	3	4	1	3	4	4	4	2	1	4	4	2
Q.No.	21	22	23	24	25	26	27	28	29	30	31									
Ans.	2	2	3	1	3	1	2	3	3	4	3									

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 4 (ANSWERS)

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	3	3	3	2	2	1	2	4	1	1	2	4	1	4	2